YouTube Lecture Links & Notes : K & I Scheme (Last Hour Revision Notes)

Topic : Statistic : Lecture 1: YT Link: https://www.youtube.com/watch?v=JN4Qpe1mctE

Topic : Derivative **(Only For K – Scheme)**: Lecture 2: YT Link: https://www.youtube.com/watch?v=vXlTGp7pk7w&list=PL3sp5tnO0dkMp4U0frcB4kfSx1npG74qZ&index=2

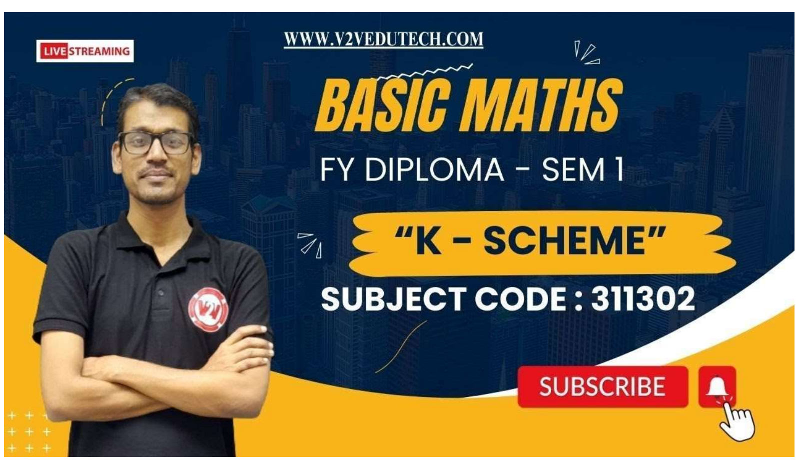
Topic : Application of Derivative **(Only For K – Scheme)** : Lecture 3: YT Link: https://www.youtube.com/watch?v=vwnTDKWNUYE

Topic : Logarithm & Partial fraction & Function : Lecture 4: YT Link: https://www.youtube.com/watch?v=P_jVW8Iqics

Topic : Straight Line : Lecture 5: YT Link: https://www.youtube.com/watch?v=l9pKCCJ81K4

Topic : Matrices : Lecture 6: YT Link: https://www.youtube.com/watch?v=791Ux-eUFpM

Topic : Revision : Lecture 7 : YT Link : https://www.youtube.com/watch?v=osaQaS4jBsU



Maths-1

K-Scheme

subject code- 311302

1) Algebra — 14 Marks
2) Trignometry — 14 Marks
3) Straight line — 8 Marks
4) Differential Calculus - 20 Marks
   - Derivative
   - Application of derivative
5) Stastics - 14 Marks

I-scheme

1) Algebra — 14 Marks
2) Trignometry — 20 Marks
3) Co-ordinate geometry — 12 Marks
4) Mensuration — 14 Marks
5) Statistics — 14 Marks

UT1 – 10/30
UT2 – 10/30

$$\frac{10+10}{2} = 10$$

$$M1 = 40 - 10$$
$$= 30$$

66
70

1) Log
2) Function
3) St. line
4) Statics
5) Derivative
6) Application of derivative

K-Scheme

10M Q1 a) log
2M b) Trignometry
2M c) Straight line
2M d) function - even or odd
2M e) Application of derivative
2M f) Application of derivative
2M g) Statiscs - Coeff. of variance

12M Q2 a) Matrices
4M b) Matrices
4M c) Partial fraction — case ①, case 2, case 3
d) Trignometry

8 Marks Q3.
a) Trignometry
b) Trignometry
4M c) St line
4M d) Derivative ⇒ $x^2+y^2+xy=8$ — find $\frac{dy}{dx}$

12 Marks Q4.
4M a) Derivative - Chain Rule
b) Derivative - (Variable) variable
4M c) Statistics - find Range & coeff. of Range
4M d) Statistics - Mean & Mean deviation
4M e) Statistics - Who is More Consistent.

12 Marks Q5.
6M a) Matrix Inverse Method
b) Trignometry
6M c) Straight line

12 Marks Q6.
6M a) Application of derivative - Maxima & Minima
6M b) Application of derivative - Radius of Curvature
c) Stastics - Std deviation, Variance & Coeff. of variance.

Internal

$\frac{12}{30} + \frac{x}{70} = \frac{40}{100}$

28

15

10 → 30

25

52 / 70

1) Log
2) Matrices
3) St. line
4) Statistics

I - Scheme

8 Marks Q1
- a) Log (2M)
- b) Matrices area of triangle (2M)
- c) Trignometry
- d) Mensuration
- e) Mensuration
- f) Statistics - Range & Co-eff of Range (2M)
- g) Statistics - coff. of variance (2M)

12 Marks Q2
- a) Matrices (4M)
- (b) Partial fraction
- c) Statistics - Variance (4M)
- d) Cramers Rule (4M)

Q3.
- a) Trignometry
- b) Trignometry
- c) Trignometry
- d) Trignometry

8 Marks Q4.
- a) Matrices (4M)
- b) Partial fraction (4M)
- c) Trignometry
- d) Trignometry
- e) Trignometry

12 Marks Q5.
- a) St. line (6M)
- b) St. line (6M)
- c) Mensuration.

12 Marks Q6.
- a) Matrix Inverse Method (6M)
- b) Statistics - Mean & Mean deviation (6M)
- c) Coeff. of Range & Who is MORE Consistent.

STATISTICS (I+K- scheme)

2 Marks

Raw-data ⇒

Q. find Range for following data & coeff. of Range

200, 210, 208, 160, 250, 290

$S = 160$ — Smallest No.
$L = 290$ — Largest No.

$$\text{Range} = L - S = 290 - 160 = 130$$

$$\text{Coeff. Range} = \frac{(L-S)}{(L+S)} = \frac{(290-160)}{(290+160)} = \frac{130}{450} = 0.2888 \simeq 0.289$$

Ungroup data

Q. find Range & coeff. of Range

x_i	5	10	15	20	25	30
f_i	2	3	7	5	7	8

⇒ S = smallest value = 5
L = largest value = 30

$$\text{Range} = L - S = 30 - 5 = 25$$

$$\text{Coeff. of Range} = \frac{(L-S)}{(L+S)} = \frac{(30-5)}{(30+5)} = \frac{25}{35} = 0.7142 \simeq 0.714$$

Continuous group data

Q. Find Range & Coefficient of Range

Marks	10-20	20-30	30-40	40-50	50-60
No. of students	6	19	34	10	18

⇒ S = smallest = 10
L = largest value = 60

$$\text{Range} = L - S = 60 - 10 = 50$$

$$\text{Coeff. Range} = \frac{(L-S)}{(L+S)} = \frac{(60-10)}{(60+10)} = \frac{50}{70} = 0.7142 \simeq 0.714$$

discontinuous group data

Q. Find Range & coeff. of Range for

Marks	10-19	20-29	30-39	40-49	50-59	60-69
No. of students	6	10	16	14	8	4

(9.5, 39.5, 49.5, 59.5, 69.5)

⇒ $\frac{(19+20)}{2} = 19.5$, $\frac{(29+30)}{2} = 29.5$

Marks	9.5-19.5	19.5-29.5	29.5-39.5	39.5-49.5	49.5-59.5	59.5-69.5
No. of students	6	10	16	14	8	4

S = smallest value = 9.5
L = largest value = 69.5

$$\text{Range} = L - S = 69.5 - 9.5 = 60$$

$$\text{Coeff. of Range} = \frac{(L-S)}{(L+S)} = \frac{(69.5-9.5)}{(69.5+9.5)} = \frac{60}{79} = 0.7594 \simeq 0.759$$

Q. Find variance & Coeff. of Variance also Mean & std. deviation

Raw data ⇒ 49, 63, 46, 59, 65, 52, 60, 54
(1, 2, 3, 4, 5, 6, 7, 8)

⇒

$$\text{Mean} = \frac{(49+63+46+59+65+52+60+54)}{8}$$

$$\bar{x} = \frac{448}{8} = 56$$

Sr. No	x_i	$d_i = \lvert x_i - \bar{x} \rvert$	d_i^2
1	49	$\lvert 49-56 \rvert = \lvert -7 \rvert = 7$	$7^2 = 49$
2	63	$\lvert 63-56 \rvert = \lvert 7 \rvert = 7$	49
3	46	$\lvert 46-56 \rvert = 10$	100
4	59	$\lvert 59-56 \rvert = 3$	9
5	65	$\lvert 65-56 \rvert = 9$	81
6	52	$\lvert 52-56 \rvert = 4$	16
7	60	$\lvert 60-56 \rvert = 4$	16
8	54	$\lvert 54-56 \rvert = 2$	4
		$\Sigma \lvert d_i \rvert = 46$	$\Sigma d_i^2 = 324$

$$S.D = \sqrt{\frac{\Sigma d_i^2}{N}} = \sqrt{\left(\frac{324}{8}\right)}$$

$= 6.3639$

$S.D \simeq 6.364 = \sigma$

$\text{Variance} = \sigma^2 = (S.D)^2$

$= 6.364^2$

$= 40.5004$

$\simeq 40.5$

$$\text{Coeff. of Variance} = \frac{S.D}{\text{Mean}} \times 100$$

$$= \frac{6.364}{56} \times 100$$

$= 11.364\,\%$

$$\text{Mean deviation} = \frac{\Sigma \lvert d_i \rvert}{N} = \frac{46}{8}$$

$= 5.75$ //

Q. Find Mean, S.D, Variance & coeff of variance

ungroup data

x	10	15	20	25
f	17	22	19	16

⇒

x_i	f	$f_i x_i$	$d_i = \lvert x_i - \bar{x} \rvert$	d_i^2	$f_i d_i^2$	$f_i \lvert d_i \rvert$
10	17	$10 \times 17 = 170$	$\lvert 10 - 17.297 \rvert = \lvert -7.297 \rvert = 7.297$	$7.297^2 = 53.246$	$17 \times 53.246 = 1007.182$	$17 \times 7.297 = 124.049$
15	22	$15 \times 22 = 330$	$\lvert 15 - 7.297 \rvert = 2.297$	$2.297^2 = 5.276$	$22 \times 5.276 = 116.072$	$22 \times 2.297 = 50.534$
20	19	$20 \times 19 = 380$	$\lvert 20 - 17.297 \rvert = 2.703$	$2.703^2 = 7.306$	$19 \times 7.306 = 138.814$	$19 \times 2.703 = 51.537$
25	16	$25 \times 16 = 400$	$\lvert 25 - 17.297 \rvert = 7.703$	$7.703^2 = 59.336$	$16 \times 59.336 = 949.376$	$16 \times 7.703 = 123.248$
	$\Sigma f = 74$	$\Sigma f_i x_i = 1280$			$\Sigma f_i d_i^2 = 2211.444$	$\Sigma f_i \lvert d_i \rvert = 349.368$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f} = \frac{1280}{74} = 17.2972$$

$$\bar{x} \simeq 17.297$$

$$S.D = \sqrt{\frac{\Sigma f_i d_i^2}{\Sigma f}} = \sqrt{\left(\frac{2211.444}{74}\right)}$$

$$S.D = 5.4666$$

$$\sigma \simeq 5.467$$

$$\text{Mean deviation} = \frac{\Sigma f_i \lvert d_i \rvert}{\Sigma f} = \frac{349.368}{74} = 4.721$$

$$\text{Variance} = S.D^2 = (5.467)^2 = 29.888$$

$$\text{Coeff. of variance} = \frac{S.D}{\text{Mean}} \times 100 = \frac{5.467}{17.297} \times 100 = 31.6066$$

$$\simeq 31.607\,\%$$

Q. Find Mean, S.D, variance & Coeff. of variance

CI	0–10	10–20	20–30	30–40	40–50
freq.	14	23	27	21	15

⇒

CI	freq.	x_i	$f_i x_i$	$d_i = \lvert x_i - \bar{x} \rvert$	d_i^2	$f_i d_i^2$
0–10	14	$\frac{0+10}{2} = 5$	$14 \times 5 = 70$	$\lvert 5-25 \rvert = \lvert -20 \rvert = 20$	$20^2 = 400$	$14 \times 400 = 5600$
10–20	23	$\frac{(10+20)}{2} = 15$	$15 \times 23 = 345$	$\lvert 15-25 \rvert = 10$	$10^2 = 100$	$23 \times 100 = 2300$
20–30	27	$\frac{(20+30)}{2} = 25$	$27 \times 25 = 675$	$\lvert 25-25 \rvert = 0$	0	$0 \times 27 = 0$
30–40	21	35	$21 \times 35 = 735$	$\lvert 35-25 \rvert = 10$	100	$21 \times 100 = 2100$
40–50	15	45	$15 \times 45 = 675$	$\lvert 45-25 \rvert = 20$	400	$15 \times 400 = 6000$
	$\Sigma f = 100$		$\Sigma f_i x_i = 2500$			$\Sigma f_i d_i^2 = 16000$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f} = \frac{2500}{100}$$

$$\bar{x} = 25$$

$$\text{S.D} = \sqrt{\frac{\Sigma f_i d_i^2}{\Sigma f}} = \sqrt{\frac{16000}{100}} = 12.6491 \simeq 12.649$$

$$\text{Variance} = \text{S.D}^2 = (12.649)^2 = 159.9972 \simeq 159.997$$

$$\text{Coeff. of Variance} = \frac{\text{S.D}}{\text{Mean}} \times 100 = \frac{12.649}{25} \times 100 = 50.596\,\%$$

Q. Find Mean, S.D, Variance & coeff. of variance

C.I	50-54	55-59	60-64	65-69
freq	3	2	3	2

⇒

49.5, 54.5, 59.5, $\frac{(64+65)}{2}$

CI	freq	Avg. CI	x_i	$f_i x_i$	$d_i = \lvert x_i - \bar{x} \rvert$	d_i^2	$f_i d_i^2$
50-54	3	49.5 – 54.5	$\frac{(49.5+54.5)}{2} = 52$	$3 \times 52 = 156$	$\lvert 52-59 \rvert = 7$	$7^2 = 49$	$3 \times 49 = 147$
55-59	2	54.5 – 59.5	$\frac{(54.5+59.5)}{2} = 57$	$2 \times 57 = 114$	$\lvert 57-59 \rvert = 2$	$2^2 = 4$	$2 \times 4 = 8$
60-64	3	59.5 – 64.5	$\frac{(59.5+64.5)}{2} = 62$	$3 \times 62 = 186$	$\lvert 62-59 \rvert = 3$	$3^2 = 9$	$3 \times 9 = 27$
65-69	2	64.5 – 69.5	$\frac{(64.5+69.5)}{2} = 67$	$2 \times 67 = 134$	$\lvert 67-59 \rvert = 8$	$8^2 = 64$	$2 \times 64 = 128$
70	$\Sigma f = 10$			$\Sigma f_i x_i = 590$			$\Sigma f_i d_i^2 = 310$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f} = \frac{590}{10} = 59$$

$$\bar{x} = 59$$

$$S.D = \sqrt{\frac{\Sigma f_i d_i^2}{\Sigma f}} = \sqrt{\left(\frac{310}{10}\right)}$$

$$= 5.5677$$

$$S.D \simeq 5.568$$

$$\text{Variance} = S.D^2$$

$$= (5.568)^2$$

$$= 31.0026$$

$$\simeq 31.003$$

$$\text{Coeff. of variance} = \frac{S.D}{\text{Mean}} \times 100$$

$$= \frac{5.568}{59} \times 100$$

$$= 9.4372$$

$$\simeq 9.437\%$$

Q. find which batsman is More consistent.

Batsman	Avg. Run	S.D
A	44	5.1
B	54	6.31

⇒ for Batsman A

Coeff. of variance = $\frac{S.D}{Mean} \times 100$

$= \frac{5.1}{44} \times 100$

$= 11.5909$

$\simeq 11.591\%$

for Batsman B

Coeff. of variance = $\frac{S.D}{Mean} \times 100$

$= \frac{6.31}{54} \times 100$

$= 11.6850$

$= 11.685\%$

Coeff. of variance of Batsman A 11.591% < Coeff. of variance of Batsman B 11.685%

$\because$ A < B

Batsman A is MORE consistent than Batsman B

14 Marks

Q. Who is More Consistent

	Worker A	Worker B
Mean time of Completing Job	40	42
Std. deviation	8	6

⇒

Coeff. of variance of Worker A $= \frac{S.D}{Mean} \times 100$

$= \frac{8}{40} \times 100$

$= 20\%$

Coeff. of variance of Worker B $= \frac{S.D}{Mean} \times 100$

$= \frac{6}{42} \times 100$

$= 14.2857$

$= 14.286\%$

Coeff. of variance of Worker A 20% > Coeff. of variance of Worker B 14.286%

A > B

∴ Worker B is More Consistent.

Derivative

1) $\frac{d}{dx}$ constant $= 0$

2) $\frac{d}{dx} x^n = n \cdot x^{n-1}$

3) $\frac{d}{dx} k x^n = k \cdot \frac{d}{dx} x^n$

4) $\frac{d}{dx} a^x = a^x \cdot \log a$

5) $\frac{d}{dx} e^x = e^x$

6) $\frac{d}{dx} \log x = \frac{1}{x}$

7) $\frac{d}{dx} \sin x = \cos x$

8) $\frac{d}{dx} \cos x = -\sin x$

9) $\frac{d}{dx} \tan x = \sec^2 x$

10) $\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$

11) $\frac{d}{dx} \sec x = \sec x \cdot \tan x$

12) $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cdot \cot x$

(*) $\frac{d}{dx}(u \pm v \mp w) = \frac{d}{dx} u \pm \frac{d}{dx} v \mp \frac{d}{dx} w$

(*) Product Rule

$$\frac{d}{dx}(u * v) = u \cdot \frac{d}{dx} v + v \cdot \frac{d}{dx} u$$

(*) Division Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{d}{dx} u - u \cdot \frac{d}{dx} v}{v^2}$$

(*) Chain Rule

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

17) $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

18) $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$

19) $\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$

20) $\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$

21) $\frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$

22) $\frac{d}{dx} \operatorname{cosec}^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}}$

find $\frac{dy}{dx}$

① $y = x^n + a^x + e^x + a^a$

differentiating w.r.t 'x'

$$\frac{d}{dx}y = \frac{d}{dx}(x^n + a^x + e^x + a^a)$$

$$\frac{dy}{dx} = \frac{d}{dx}x^n + \frac{d}{dx}a^x + \frac{d}{dx}e^x + \frac{d}{dx}a^a$$

$$\frac{dy}{dx} = n \cdot x^{n-1} + a^x \cdot \log_e a + e^x + 0 \quad //$$

⊛ $y = e^x \tan x$ (u = e^x, v = $\tan x$)

differentiating w.r.t x

$$\frac{d}{dx}y = \frac{d}{dx}(u \cdot v)$$

$$= u \cdot \frac{d}{dx}v + v \cdot \frac{d}{dx}u$$

$$= e^x \cdot \frac{d}{dx}\tan x + \tan x \cdot \frac{d}{dx}e^x$$

$$\frac{dy}{dx} = e^x \cdot \sec^2 x + \tan x \cdot e^x \quad //$$

$$\frac{dy}{dx} = e^x(\sec^2 x + \tan x) \quad //$$

⊛ if $y = \frac{a^x}{\sin x}$ (u = a^x, v = $\sin x$)

differentiating w.r.t x

$$\frac{d}{dx}y = \frac{d}{dx}\left(\frac{u}{v}\right)$$

$$= \frac{v \cdot \frac{d}{dx}u - u \cdot \frac{d}{dx}v}{v^2} = \frac{\sin x \cdot \frac{d}{dx}a^x - a^x \cdot \frac{d}{dx}\sin x}{(\sin x)^2}$$

$$\frac{dy}{dx} = \frac{\sin x \cdot a^x \cdot \log_e a - a^x \cdot \cos x}{(\sin x)^2} \quad //$$

$$\frac{d}{dx} a^{5x} = \frac{d}{dx} a^{x}$$
$$= a^{x} \cdot \log_e a \cdot \frac{d}{dx} x$$
$$= a^{5x} \cdot \log_e a \cdot \frac{d}{dx} 5x$$
$$= a^{5x} \cdot \log_e a \cdot 5\left(\frac{d}{dx} x^1\right)$$
$$= a^{5x} \cdot \log_e a \cdot 5(1)$$
$$= 5 \cdot a^{5x} \cdot \log_e a$$

$n \cdot x^{n-1}$
$1 \cdot x^{1-1}$
$1 \, x^{0}$
$1 \times 1 = 1$

$$\frac{d}{dx} \sin(6x) = \frac{d}{dx} \sin x$$
$$= \cos x \, \frac{d}{dx} x$$
$$= \cos 6x \cdot \frac{d}{dx} 6x$$
$$= \cos 6x \cdot 6\left(\frac{d}{dx} x\right)$$
$$= 6 \cdot \cos 6x$$

$$\frac{d}{dx} \sin^6 x = \frac{d}{dx} (\sin x)^6$$
$$= \frac{d}{dx} x^6 = 6x^{6-1} \frac{d}{dx} x$$
$$= 6(\sin x)^5 \cdot \frac{d}{dx} \sin x$$
$$= 6 \cdot (\sin x)^5 \cdot (\cos x)$$

$$\frac{d}{dx} e^{7x} = \frac{d}{dx} e^{x}$$
$$= e^{x} \frac{d}{dx} x$$
$$= e^{7x} \cdot \frac{d}{dx} 7x$$
$$= e^{7x} \cdot 7\left(\frac{d}{dx} x\right)$$
$$= e^{7x} \cdot 7(1)$$

* $y = x^5 + a^{6x} + e^{7x} + a^8 + 3x^4$

differentiating w.r.t 'x'

$$\frac{d}{dx} y = \frac{d}{dx}\left(x^5 + a^{6x} + e^{7x} + a^8 + 3x^4\right)$$
$$= \frac{d}{dx} x^5 + \frac{d}{dx} a^{6x} + \frac{d}{dx} e^{7x} + \frac{d}{dx} a^8 + \frac{d}{dx} 3x^4$$
$$= 5x^4 + a^{x} \cdot \log_e a \cdot \frac{d}{dx} x + e^{x} \cdot \frac{d}{dx} x + 0 + 3 \cdot \frac{d}{dx} x^4$$
$$= 5x^4 + a^{6x} \cdot \log_e a \cdot \frac{d}{dx} 6x + e^{7x} \cdot \frac{d}{dx} 7x + 3(4 \cdot x^3)$$
$$\frac{dy}{dx} = 5x^4 + a^{6x} \cdot \log_e a \cdot 6(1) + e^{7x} \cdot 7(1) + 12x^3$$

$a(b \times d)$ → $a \times b \times d$

$a(b + d)$ → $a \times b + a \times d$

$a(b - d)$ → $a \times b - a \times d$

⊛ $y = e^{6x} \cdot \tan(3x)$ ($u = e^{6x}$, $v = \tan(3x)$)

differentiating w.r.t x

$$\frac{d}{dx} y = \frac{d}{dx}(u \cdot v)$$
$$= u \cdot \frac{d}{dx} v + v \cdot \frac{d}{dx} u$$
$$= e^{6x} \cdot \frac{d}{dx} \tan(3x) + \tan(3x) \cdot \frac{d}{dx} e^{6x}$$
$$= e^{6x} \cdot \sec^2 x \, \frac{d}{dx} x + \tan(3x) \cdot e^{x} \frac{d}{dx} x$$
$$= e^{6x} \cdot \sec^2(3x) \cdot \frac{d}{dx} 3x + \tan(3x) \cdot e^{6x} \cdot \frac{d}{dx} 6x$$
$$= e^{6x} \cdot \sec^2(3x) \cdot 3(1) + \tan(3x) \cdot e^{6x} \cdot 6(1)$$

Home work

⊛ $y = \dfrac{a^{6x}}{\sin(5x)}$

$(\text{variable})^{\text{constant}}$ $\qquad \frac{d}{dx} x^n = n \cdot x^{n-1}$

$(\text{constant})^{\text{variable}}$ $\qquad \frac{d}{dx} a^x = a^x \cdot \log_e a$

$(\text{constant})^{\text{constant}}$ $\qquad \frac{d}{dx} a^a = 0$

$y = x^x$ $\quad \rightarrow (\text{variable})^{\text{variable}}$

taking 'log' on both sides

$\log y = \log x^x$

$(\log y) = x(\log x)$

differentiating w.r.t x

$\frac{d}{dx} \log y = \frac{d}{dx} (x \cdot \log x)$

$(u \cdot v)$

$\frac{1}{x} \frac{d}{dx} x = u \cdot \frac{d}{dx} v + v \cdot \frac{d}{dx} u$

$\frac{1}{y} \cdot \left(\frac{d}{dx} y\right) = x \cdot \left(\frac{d}{dx} \log x\right) + \log x \cdot \left(\frac{d}{dx} x\right)$

$\frac{1}{y} \left(\frac{dy}{dx}\right) = x \cdot \frac{1}{x} + \log x \cdot (1)$

$\frac{1}{y} \left(\frac{dy}{dx}\right) = 1 + \log x$

$\frac{dy}{dx} = y(1 + \log x)$ //

$$\log (a \times b) = \log a + \log b$$

$$\log \left(\frac{a}{b}\right) = \log a - \log b$$

$$\log_a a = 1$$

$$\log_x 1 = 0$$

$$\log_e a^{x} = x . \log_e a$$

$$y = x^{\sin x}$$

taking log on both sides

$$\log y = \log x^{\sin x}$$

$$\log y = \sin x \cdot \log x$$

differentiating w.r.t x

$$\frac{d}{dx} \log y = \frac{d}{dx} (\underset{u}{\sin x} \cdot \underset{v}{\log x})$$

$$\frac{1}{X} \cdot \frac{d}{dx} X = \frac{d}{dx} (u \cdot v)$$

$$\frac{1}{y} \cdot \frac{d}{dx} y = u \cdot \frac{d}{dx} v + v \cdot \frac{d}{dx} u$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \sin x \cdot \frac{d}{dx} \log x + \log x \cdot \frac{d}{dx} \sin x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \sin x \cdot \frac{1}{x} + \log x \cdot \cos x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{\sin x}{x} + \log x \cdot \cos x$$

$$\frac{dy}{dx} = y \left[\frac{\sin x}{x} + \log x \cdot \cos x \right] //$$

$$y = x^{\sin x} + (\tan x)^x$$

$$y = u + v$$

diff. w.r.t x

$$\frac{d}{dx} y = \frac{d}{dx}(u+v)$$

$$= \frac{d}{dx} u + \frac{d}{dx} v$$

$$u = x^{\sin x}$$

taking log on both side

$$\log u = \log x^{\sin x}$$

$$\log u = \sin x \cdot \log x$$

diff. w.r.t x

$$\frac{d}{dx} \log u = \frac{d}{dx}(\sin x \cdot \log x)$$

$$\frac{1}{X} \frac{d}{dx} X = \frac{d}{dx}(U \cdot V)$$

$$\frac{1}{u} \cdot \frac{d}{dx} u = U \cdot \frac{d}{dx} V + V \cdot \frac{d}{dx} U$$

$$= \sin x \cdot \frac{d}{dx} \log x + \log x \cdot \frac{d}{dx} \sin x$$

$$\frac{1}{u} \cdot \frac{du}{dx} = \sin x \cdot \frac{1}{x} + \log x \cdot \cos x$$

$$\frac{du}{dx} = u\left[\sin x \cdot \frac{1}{x} + \log x \cdot \cos x\right]$$

$$\frac{du}{dx} = x^{\sin x}\left[\sin x \cdot \frac{1}{x} + \log x \cdot \cos x\right]$$

$$v = (\tan x)^x$$

taking log on both sides

$$\log v = \log (\tan x)^x$$

$$\log v = x \cdot \log(\tan x)$$

diff. w.r.t x

$$\frac{d}{dx} \log v = \frac{d}{dx}\left[x \cdot \log(\tan x)\right]$$

$$\frac{1}{X} \cdot \frac{d}{dx} X = \frac{d}{dx}(U \cdot V)$$

$$\frac{1}{v} \cdot \frac{d}{dx} v = U \cdot \frac{d}{dx} V + V \cdot \frac{d}{dx} U$$

$$= x \cdot \frac{d}{dx} \log(\tan x) + \log(\tan x) \cdot \frac{d}{dx} x$$

$$= x \cdot \frac{1}{X} \frac{d}{dx} X + \log(\tan x)(1)$$

$$= x \cdot \frac{1}{\tan x} \cdot \frac{d}{dx} \tan x + \log(\tan x)$$

$$\frac{1}{v} \frac{dv}{dx} = x \cdot \frac{1}{\tan x} \cdot \sec^2 x + \log(\tan x)$$

$$\frac{dv}{dx} = v\left[x \cdot \frac{1}{\tan x} \cdot \sec^2 x + \log(\tan x)\right]$$

$$\frac{dv}{dx} = (\tan x)^x\left[x \cdot \frac{1}{\tan x} \cdot \sec^2 x + \log(\tan x)\right]$$

$$\frac{dy}{dx} = \frac{d}{dx} u + \frac{d}{dx} v$$

$$= x^{\sin x}\left[\sin x \cdot \frac{1}{x} + \log x \cdot \cos x\right] + (\tan x)^x\left[x \cdot \frac{1}{\tan x} \cdot \sec^2 x + \log(\tan x)\right] //$$

$x = a(\theta + \sin\theta)$

$y = a(1 - \cos\theta)$

find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$

$\Rightarrow$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$y = a(1-\cos\theta)$

differenting w.r.t θ

$$\frac{d}{d\theta} y = \frac{d}{d\theta} a(1-\cos\theta)$$

$$= a \cdot \frac{d}{d\theta}(1-\cos\theta)$$

$$= a\left[\frac{d}{d\theta} 1 - \frac{d}{d\theta}\cos\theta\right]$$

$$\frac{dy}{d\theta} = a\left[0 - (-\sin\theta)\right]$$

$x = a(\theta + \sin\theta)$

differentiating w.r.t θ

$$\frac{d}{d\theta} x = \frac{d}{d\theta} a(\theta + \sin\theta)$$

$$= a \cdot \frac{d}{d\theta}(\theta + \sin\theta)$$

$$= a \cdot \left[\frac{d}{d\theta}\theta + \frac{d}{d\theta}\sin\theta\right]$$

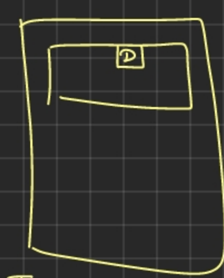
$$\frac{dx}{d\theta} = a\left[1 + \cos\theta\right]$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a[0-(-\sin\theta)]}{a[1+\cos\theta]}$$

$$= \frac{a[0-(-\sin 90)]}{a[1+\cos 90]}$$

$$= \frac{a \times 1}{a[1]}$$

$$\frac{dy}{dx} = 1$$



$\theta = \frac{\pi}{2}$

$\pi = 180°$

$\frac{\pi}{2} = \frac{180}{2} = 90°$

Application of Derivative

$\frac{d}{dx}$ constant $= 0$

$\frac{d}{dx} x^n = n \cdot x^{n-1}$

$\frac{d}{dx} kx^n = k \cdot \frac{d}{dx} x^n$

$\frac{d}{dx} a^x = a^x \cdot \log_e a$

$\frac{d}{dx} e^x = e^x$

$\frac{d}{dx} \log x = \frac{1}{x}$

$\frac{d}{dx} \sin x = \cos x$

$\frac{d}{dx} \cos x = -\sin x$

$\frac{d}{dx} \tan x = \sec^2 x$

$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$

$\frac{d}{dx} \sec x = \sec x \cdot \tan x$

$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cdot \cot x$

$$\frac{d}{dx}(u \pm v \mp w) = \left[\frac{d}{dx} u \pm \frac{d}{dx} v \pm \frac{d}{dx} w\right]$$

$$\frac{d}{dx}(u \cdot v) = u \cdot \frac{d}{dx} v + v \cdot \frac{d}{dx} u$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{d}{dx} u - u \frac{d}{dx} v}{v^2}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

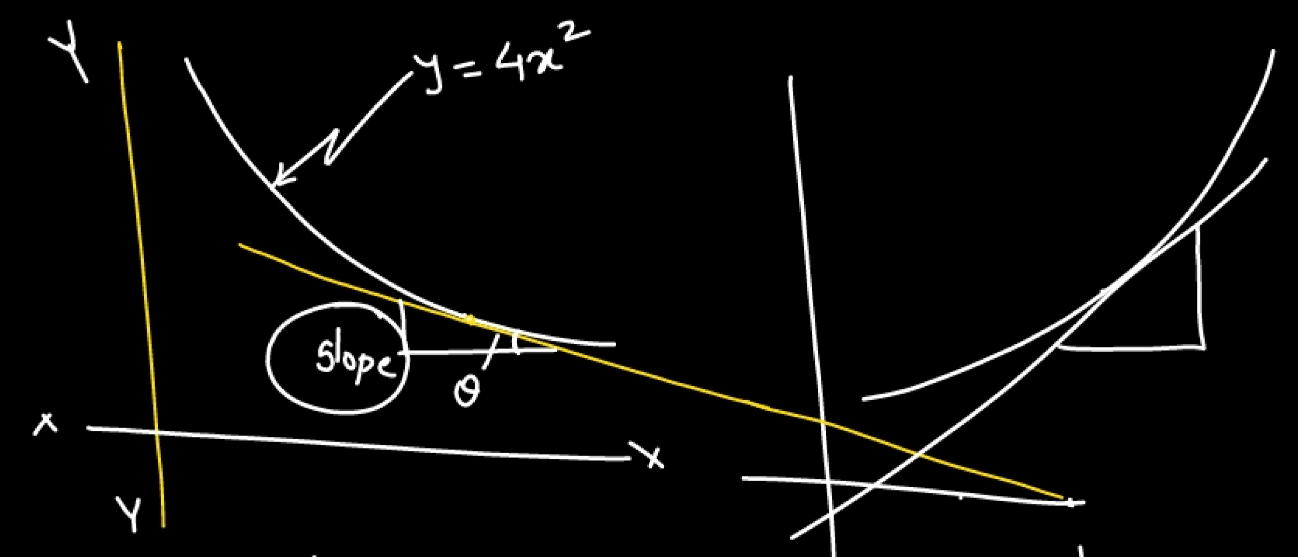
$\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$

$\frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$

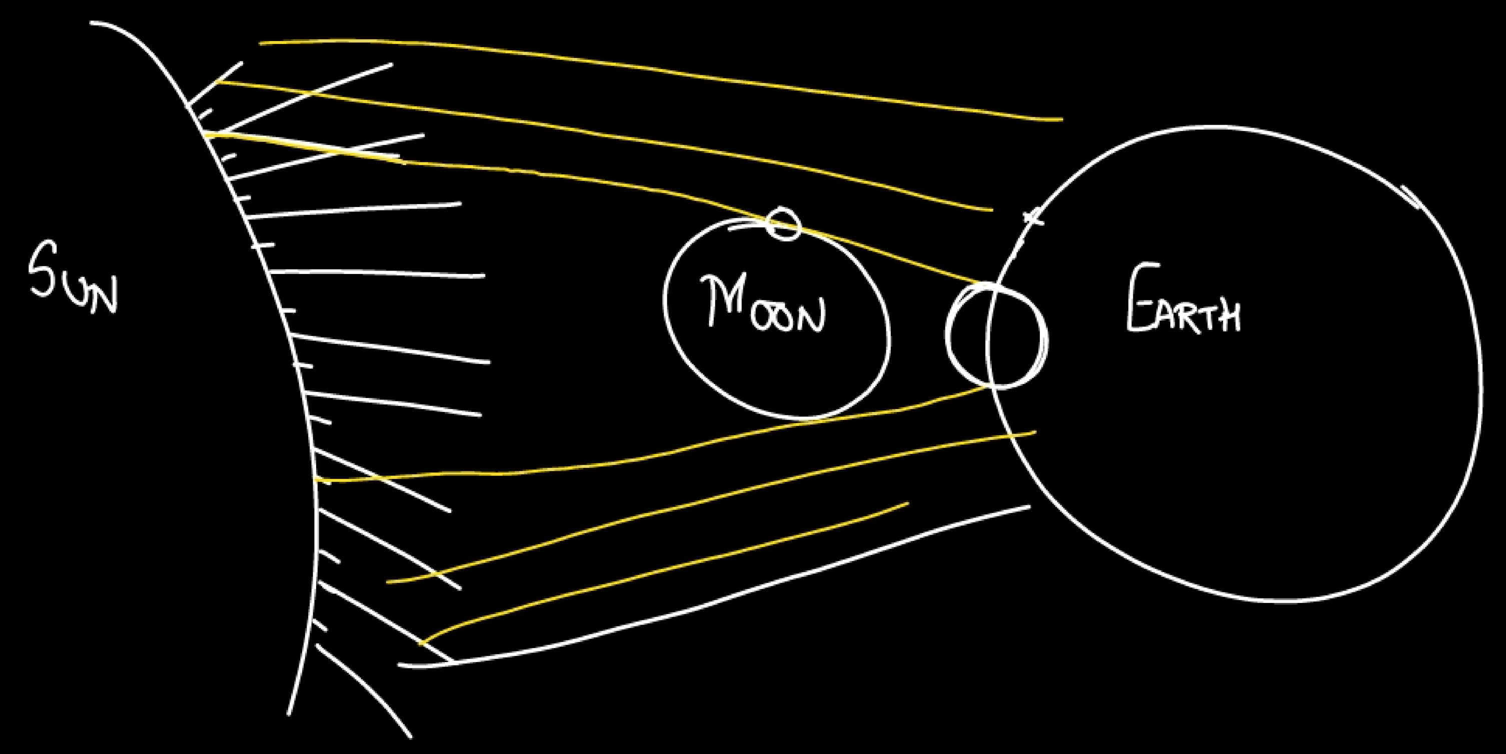
$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$

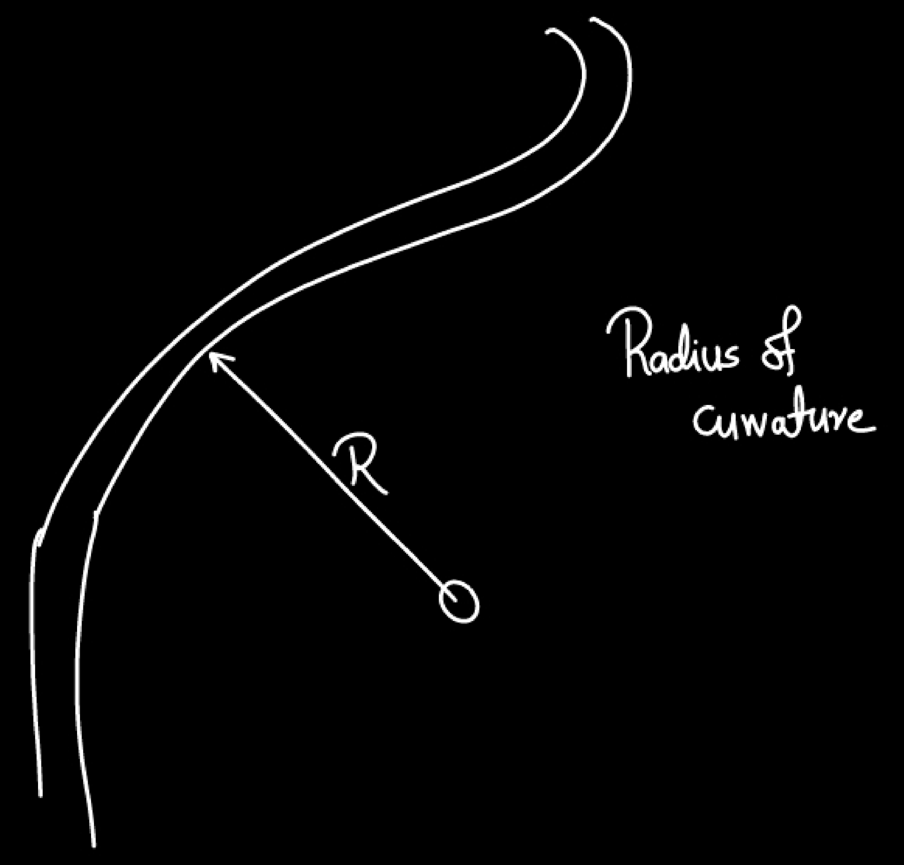
$\frac{d}{dx} \operatorname{cosec}^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}}$



$$\text{slope} = \theta = \tan\theta = \frac{dy}{dx}$$

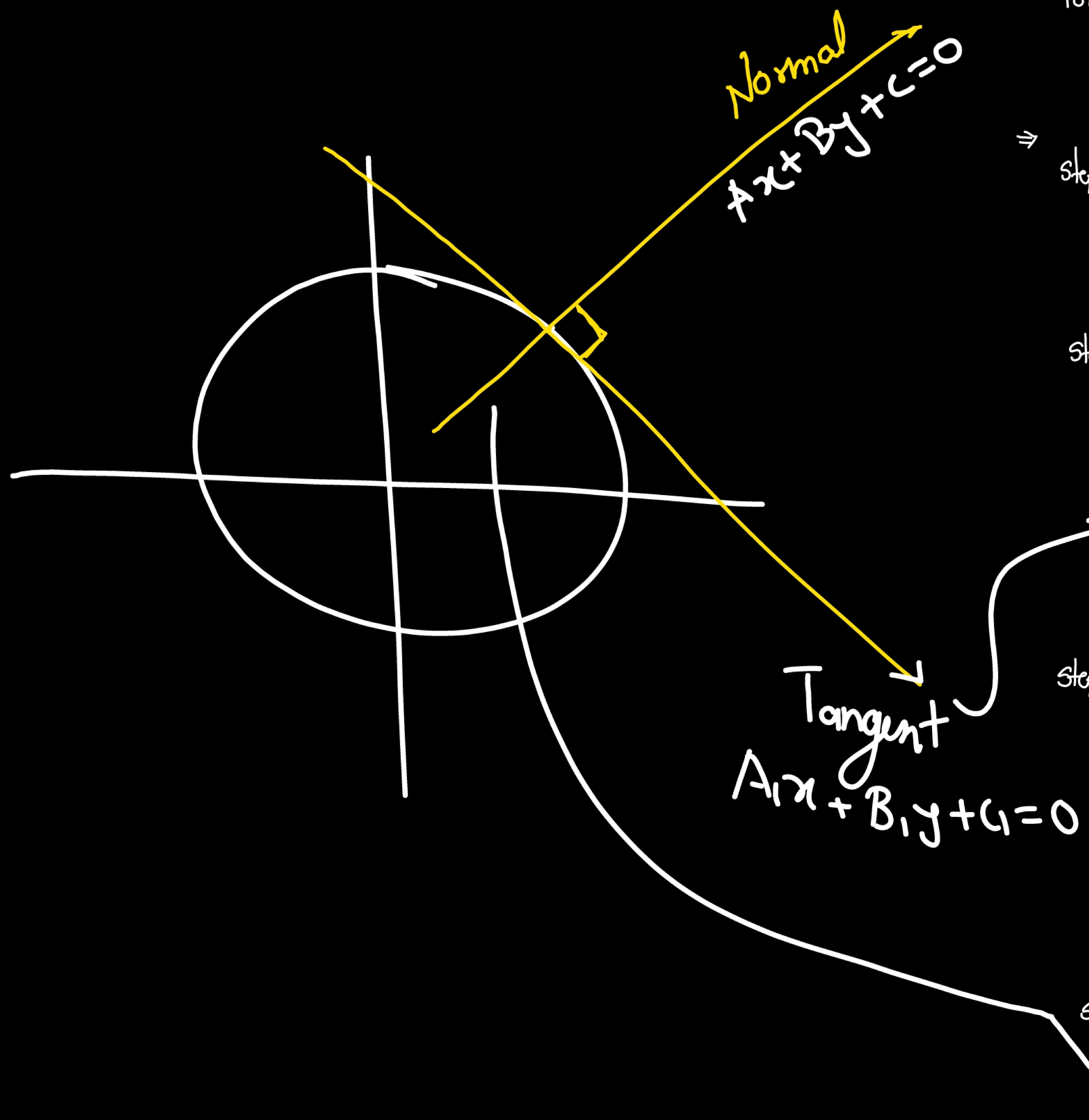
θ ↓ w.r.t x-axis





$$y = 5x - x^2 + x^3$$

Maxima & Minima



For finding eqⁿ of Tangent & Normal

Give ⇒ Eqⁿ of curve $= y$
Point (x_1, y_1)

⇒ Step ① Find slope of tangent (m_1)

find $\left(\frac{dy}{dx}\right)\Big|_{at\ (x_1, y_1)} = m_1$

Step ② Equation of Tangent
a) Point (x_1, y_1)
b) slope $= m_1$

∴ Eqⁿ of Tangent

$(y-y_1) = m_1(x-x_1)$

Step ③ Find slope of Normal (m_2)

slope of tangent × slope of Normal $= -1$

$m_1 \times m_2 = -1$ (from step 1)

$m_2 =$ slope of Normal

Step ④ Eqⁿ of Normal
a) Point (x_1, y_1)
b) slope $= m_2$

Eqⁿ of Normal.

$(y-y_1) = m_2(x-x_1)$

Application of derivative for —

Q. Find eqⁿ of Tangent & Normal to curve $4x^2 + 9y^2 = 40$ at $(3,2)$

⇒

Step ① Find slope of tangent (m_1)

$$4x^2 + 9y^2 = 40$$

differentiating w.r.t x

$$\frac{d}{dx}(4x^2 + 9y^2) = \frac{d}{dx}40$$

$$\frac{d}{dx}4x^2 + \frac{d}{dx}9y^2 = 0$$

$$4\frac{d}{dx}x^2 + 9\frac{d}{dx}y^2_x = 0$$

$$4(2x) + 9\left(2X \cdot \frac{d}{dx}x\right) = 0$$

$$8x + 9\left(2y \cdot \frac{dy}{dx}\right) = 0$$

$$18y\frac{dy}{dx} = -8x$$

$$\frac{dy}{dx} = \frac{-8x}{18y}$$

$$m_1 = \left.\frac{dy}{dx}\right|_{at(3,2)} = \frac{(-8\times3)}{(18\times2)} = \frac{-24}{+36}$$

(x = 3, y = 2)

$$\therefore m_1 = -0.666(7)$$

$$m_1 = -0.667$$

Step ② Find eqⁿ of Tangent
- a) Point $(x_1, y_1) = (3,2)$
- b) slope $= m_1 = -0.667$

Eqⁿ of tangent

$$(y - y_1) = m_1(x - x_1)$$

$$(y-2) = -0.667(x-3)$$

$$y - 2 = x(-0.667) - 3(-0.667)$$

$$+y - 2 = -0.667x + 2.001$$

$$+0.667x - 2.001 + y - 2 = 0$$

$$0.667x + y - 4.001 = 0$$

↳ eqⁿ of tangent

Step ③ Find slope of Normal (m_2)

slope of tangent × slope of Normal = −1

$$m_1 \times m_2 = -1$$

$$\therefore -0.667 \times m_2 = -1$$

$$\therefore m_2 = \frac{-1}{-0.667} = 1.499$$

Step ④ - Find eqⁿ of Normal
- a) Point $= (x_1, y_1) = (3,2)$
- b) Slope $= m_2 = 1.499$

Eqⁿ of Normal

$$(y - y_1) = m_2(x - x_1)$$

$$(y-2) = 1.499(x-3)$$

$$(y-2) = 1.499x - 1.499\times3$$

$$+y - 2 = 1.499x - 4.497$$

$$-1.499x + 4.497 + y - 2 = 0$$

$$-1.499x + y + 2.497 = 0$$

↳ eqⁿ of Normal

Q. At which point on curve

$y = 3x - x^2$ the slope of tangent is -5?

$\Rightarrow$

$$y = 3x - x^2$$

differentiating w.r.t x

$$\frac{d}{dx} y = \frac{d}{dx}(3x - x^2)$$

$$\frac{dy}{dx} = \frac{d}{dx} 3x - \frac{d}{dx} x^2$$

$$\frac{dy}{dx} = 3\frac{d}{dx}x - 2x$$

$$= 3(1) - 2x$$

$$\frac{dy}{dx} = 3 - 2x = \text{slope of tangent}$$

$$3 - 2x = -5$$

$$3 + 5 = 2x$$

$$8 = 2x$$

$$\frac{8}{2} = x$$

$$\boxed{x = 4}$$

from eq^n of curve

$$y = 3x - x^2$$

$$= 3 \times 4 - 4^2$$

$$\boxed{y = -4}$$

$\therefore$ Point $(x_1, y_1) = (4, -4)$ //

Radius of Curvature

$$(\rho) = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

Step ① Find $\left(\frac{dy}{dx}\right)$

step ② find $\left(\frac{d^2y}{dx^2}\right)$

step ③ find $(\rho) = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ units //

Q. find Radius of Curvature to curve $y = x^5$ at $(3, -3)$

⇒

step ① find $\frac{dy}{dx}$

$$y = x^5$$

differentiating w.r.t x

$$\frac{d}{dx}y = \frac{d}{dx}x^5$$

$$\frac{dy}{dx} = 5x^4 \quad \text{——— ①}$$

$$\left.\frac{dy}{dx}\right|_{at\,(3,-3)} = 5(3)^4$$

($x = 3$, $y = -3$)

$$= 405$$

step ② find $\frac{d^2y}{dx^2}$

from eqⁿ ① $\frac{dy}{dx} = 5x^4$

differentiating w.r.t 'x'

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}5x^4$$

$$\frac{d^2y}{dx^2} = 5 \cdot \frac{d}{dx}x^4$$

$$\frac{d^2y}{dx^2} = 5(4x^3)$$

$$\frac{d^2y}{dx^2} = 20x^3$$

$$\left.\frac{d^2y}{dx^2}\right|_{at\,(3,-3)} = 20 \times 3^3$$

($x = 3$, $y = -3$)

$$= 540$$

step ③ Find Radius of Curvature

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$= \frac{\left[1 + (405)^2\right]^{1.5}}{540}$$

$$= \frac{(164026)^{1.5}}{540}$$

$$= \frac{66430732.5}{540}$$

$$\rho = 123019.875 \text{ units}$$

Q. A beam is bent in form of curve

$y = 2\sin x - \sin 2x$

find Radius of curvature at $x = \frac{\pi}{2}$

⇒ Step ①: Find $\frac{dy}{dx}$

$\pi = 180°$

$\frac{\pi}{2} = \frac{180°}{2} = 90°$

$y = 2\sin x - \sin 2x$

differentiating w.r.t x

$\frac{d}{dx} y = \frac{d}{dx}(2\sin x - \sin 2x)$

$= \frac{d}{dx} 2\sin x - \frac{d}{dx}\sin 2x$

$= 2\frac{d}{dx}\sin x - \cos X \cdot \frac{d}{dx} X$

$= 2\cos x - \cos(2x) \cdot \frac{d}{dx} 2x$

$\frac{dy}{dx} = 2\cos x - \cos(2x) \cdot 2$ ——— ①

$\left.\frac{dy}{dx}\right|_{at\ x=\pi/2,\ x=90°} = 2\cos 90 - \cos(2\times 90) \cdot 2$

$= 2\times 0 - \cos(180)\times 2$

$= 0 - (-1)\times 2$

$\left.\frac{dy}{dx}\right|_{x=\pi/2} = 2$

Step ② find $\frac{d^2y}{dx^2}$

eqⁿ ① ⇒ $\frac{dy}{dx} = 2\cos x - \cos(2x) \cdot 2$

differentiating w.r.t x

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}[2\cos x - \cos(2x) \cdot 2]$

$\frac{d^2y}{dx^2} = \frac{d}{dx} 2\cos x - \frac{d}{dx}\cos(2x) \cdot 2$

$= 2\frac{d}{dx}\cos x - 2\frac{d}{dx}\cos(2x)$

$= 2(-\sin x) - 2(-\sin X) \cdot \frac{d}{dx} X$

$= 2(-\sin x) - 2[-\sin(2x)] \cdot \frac{d}{dx} 2x$

$\frac{d^2y}{dx^2} = 2(-\sin x) - 2[-\sin(2x)] \cdot 2(1)$

$\frac{\pi}{2} = \frac{180°}{2} = 90°$

$\left.\frac{d^2y}{dx^2}\right|_{at\ x=\pi/2,\ 90°} = 2(-\sin 90) - 2[-\sin(2\times 90)]\times 2$

$= 2(-1) - 2[0]\times 2$

$\left.\frac{d^2y}{dx^2}\right|_{at\ x=\pi/2} = -2$

Step ③ Find Radius of Curvature

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$\rho = \frac{[1 + 2^2]^{1.5}}{(-2)}$$

$$= \frac{5^{1.5}}{(-2)}$$

$$= \frac{11.180}{(-2)}$$

$\rho = -5.59$ units

$\pi = 180°$

$\frac{\pi}{4} = \frac{180°}{4} = 45°$

$\frac{\pi}{3} = \frac{180}{3} = 60°$

Maxima & Minima

eqn

Step ① Find value of $\left(\frac{dy}{dx}\right)$

Step ② Find value of x if $\frac{dy}{dx} = 0$

x_1 & x_2

Step ③ Find value of $\frac{d^2y}{dx^2}$

Step ④ Find value of

$\left.\frac{d^2y}{dx^2}\right\|_{at\ x=x_1} =$	$\left.\frac{d^2y}{dx^2}\right\|_{at\ x=x_2} =$
$= \_\_\_ < 0$	$= \_\_\_ > 0$
$\because \frac{d^2y}{dx^2}$ is Negative	$\because \frac{d^2y}{dx^2}$ is positive
$\therefore$ function is Maxima	$\therefore$ function is Minima

Step ⑤

at $x = x_1$	at $x = x_2$
$y_{max} = \_\_\_\_\_$	$y_{min} = \_\_\_\_\_$

Q. Find Maxima & Minima for

$$y = x^3 - 9x^2 + 24x$$

⇒ step ① find value of $\frac{dy}{dx}$

$$y = x^3 - 9x^2 + 24x$$

differentiating w.r.t x

$$\frac{d}{dx} y = \frac{d}{dx}\left(x^3 - 9x^2 + 24x\right)$$

$$= \frac{d}{dx} x^3 - \frac{d}{dx} 9x^2 + \frac{d}{dx} 24x$$

$$= 3x^2 - 9\frac{d}{dx} x^2 + 24\frac{d}{dx} x$$

$$\frac{dy}{dx} = 3x^2 - 9(2x) + 24(1) \quad \text{——— ①}$$

step ② find value of x if $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 0 = 3x^2 - 9(2x) + 24$$

$$0 = 3x^2 - 18x + 24$$

991 MS

EQN

1

unknown?  degree?

2 3 — 2 ✓ 3

a? 3

b? -18

c? 24

$x_1 = 4$

$x_2 = 2$

991 ES

Mode

5 EQN

1 $a_n x + b_n y = c_n$

2 $a_n x + b_n y + c_n z = d_n$

✓3 $ax^2 + bx + c = 0$

4 $ax^3 + bx^2 + cx + d = 0$

$a = 3$ $c = 24$

$b = -18$

step ③ Find value of $\frac{d^2y}{dx^2}$

eqⁿ ① ⇒ $\frac{dy}{dx} = 3x^2 - 9(2x) + 24$

differentiating w.r.t x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(3x^2 - 18x + 24\right)$$

$$= \frac{d}{dx} 3x^2 - \frac{d}{dx} 18x + \frac{d}{dx} 24$$

$$\frac{d^2y}{dx^2} = 3\frac{d}{dx} x^2 - 18\frac{d}{dx} x + 0$$

$$= 3(2x) - 18(1)$$

$$\frac{d^2y}{dx^2} = 6x - 18$$

step ④ $x_1 = 4$

$$\left.\frac{d^2y}{dx^2}\right|_{\text{at } x = x_1,\ x = 4} = 6x - 18$$

$$= 6 \times 4 - 18$$

$$= 6 > 0$$

∴ $\frac{d^2y}{dx^2}$ is positive

∴ function is Minima

$x_2 = 2$

$$\left.\frac{d^2y}{dx^2}\right|_{\text{at } x = x_2,\ x = 2} = 6x - 18$$

$$= 6 \times 2 - 18$$

$$= -6 < 0$$

∴ $\frac{d^2y}{dx^2}$ is negative

∴ function is Maxima

step ⑤ $x_1 = 4$

$$y_{min} = x^3 - 9x^2 + 24x$$

$$= 4^3 - 9 \times 4^2 + 24 \times 4$$

$$y_{min} = 16$$

$x_2 = 2$

$$y_{max} = x^3 - 9x^2 + 24x$$

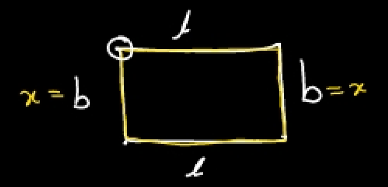
$$= 2^3 - 9 \times 2^2 + 24 \times 2$$

$$y_{max} = 20$$

Q. A metal wire 100 cm long bent to form a rectangle find its dimension if area is Maximum.

⇒

step ①



Area = $l \times b$

$y = l \times b$

$y = (50 - x)\, x$

$y = 50x - x^2$

$100 = l + b + l + b$

$= 2l + 2b$

$100 = 2(l + b)$

$\frac{100}{2} = l + b = 50$

$l + x = 50$

$l = 50 - x$

$y = 50x - x^2$

step ② find value of $\frac{dy}{dx}$

$y = 50x - x^2$

differentiating w.r.t x

$\frac{d}{dx} y = \frac{d}{dx}(50x - x^2)$

$= \frac{d}{dx} 50x - \frac{d}{dx} x^2$

$\frac{dy}{dx} = 50 \frac{d}{dx} x - 2x$

$\frac{dy}{dx} = 50 - 2x$ ——— ①

step ③ : find value of x if $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 50 - 2x = 0$

$50 = 2x$

$\frac{50}{2} = x$

$\boxed{25 = x}$

step ④ : Find value of $\frac{d^2y}{dx^2}$

$\frac{dy}{dx} = 50 - 2x$

differentiating w.r.t x

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(50 - 2x)$

$\frac{d^2y}{dx^2} = \frac{d}{dx} 50 - \frac{d}{dx} 2x$

$= 0 - 2 \frac{d}{dx} x$

$= 0 - 2 \times 1$

$\frac{d^2y}{dx^2} = -2 < 0$

$\because \frac{d^2y}{dx^2}$ is Negative

function is Maxima

step ⑤ : $x = 25$

$y_{max} = 50x - x^2$

$= 50 \times 25 - 25^2$

$y_{max} = 625$ sq.cm

$b = x = 25$ cm

$l = 50 - x = 50 - 25$

$= 25$ cm //

Q. Manufacturer can sell 'x' items at prise of ₹ $(330-x)$ each. Cost of production of x items is ₹ $(x^2+10x+12)$. Determin No. of items to be sold so that manufacturer can make max. profit

⇒

Profit = No. of items × prise of item − cost of production

$$y = x(330-x) - (x^2+10x+12)$$

$$y = 330x - x^2 - x^2 - 10x - 12$$

$$y = 320x - x^2 - 12$$

step ① find value of $\frac{dy}{dx}$

$$y = 320x - x^2 - 12$$

diff. w.r.t x

$$\frac{d}{dx}y = \frac{d}{dx}(320x - x^2 - 12)$$

$$= \frac{d}{dx}320x - \frac{d}{dx}x^2 - \frac{d}{dx}12$$

$$= 320\frac{d}{dx}x - 2x - 0$$

$$\frac{dy}{dx} = 320 - 2x \quad \text{—— ①}$$

step ② find value of x if $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 320 - 2x = 0$$

$$320 = 2x$$

$$\frac{320}{2} = x$$

$$\therefore \boxed{x = 160}$$

step ③ find value of $\frac{d^2y}{dx^2}$

from ① $\frac{dy}{dx} = 320 - 2x$

diff. w.r.t x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(320-2x)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}320 - \frac{d}{dx}2x$$

$$= 0 - 2\frac{d}{dx}x$$

$$= 0 - 2(1)$$

$$\frac{d^2y}{dx^2} = -2 < 0$$

$\because \frac{d^2y}{dx^2}$ is Negative

function is Maxima

Step ④ $x = 160$

$$\text{Profit} = y_{max} = 320x - x^2 - 12$$

$$= 320 \times 160 - 160^2 - 12$$

$$\text{Profit} = ₹\ 25{,}588/\text{—}$$

$\therefore$ No. of items sold = 160

Logarithm

$$* \quad y = a^x$$

$$x = \log_a y$$

1) $\log(a \times b) = \log a + \log b$

2) $\log\left(\frac{a}{b}\right) = \log a - \log b$

3) $\log_a a = 1$

4) $\log_x 1 = 0$

5) $\log_e a^x = x \cdot \log_e a$

6) $a^{\log_a y} = y$

Q. Find value of $\log_2 16$

$\Rightarrow$

$\log_2 16$

$= \log_2 2^4$

$= 4 \cdot \log_2 2$

$= 4(1)$

$= 4$

$16 \Rightarrow \frac{16}{2} = 8$

$\frac{8}{2} = 4$

$\frac{4}{2} = 2$

$\frac{2}{2} = 1$

$2^4 = 16$

Q. Find value of $\log_5 125$

$\Rightarrow$

$\log_5 125$

$= \log_5 5^3$

$= 3 \cdot \log_5 5$

$= 3(1)$

$= 3$

$\frac{125}{5} = 25$

$\frac{25}{5} = 5$

$\frac{5}{5} = 1$

$5^3 = 125$

Q. Find value of $25^{\log_5 8}$

$\Rightarrow 25^{\log_5 8} = (5^2)^{\log_5 8}$

$= 5^{2 \times \log_5 8}$

$= 5^{\log_5 8^2}$

$= 8^2$

$= 64$ //

$\log a^x = x \cdot \log a$

$a^{\log_a y} = y$

$(a^m)^n = a^{m \times n}$

$a^m \times a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$

Q. Find value of

$625^{\log_5 3}$

$= 81$ //

Q. $\log_2 14 - \log_2 7$

$\Rightarrow \quad = \log_2 \left(\frac{14}{7}\right)$ —— $\log\left(\frac{a}{b}\right) = \log a - \log b$

$= \log_2 2$ —— $\log_a a = 1$

$= 1$

Q. $\log_3 4 \times \log_4 81$

$= \frac{\log 4}{\log 3} \times \frac{\log 81}{\log 4}$ —— $\log_n m = \frac{\log_e m}{\log_e n}$

$\frac{81}{3} = 27$, $\frac{27}{3} = 9$, $\frac{9}{3} = 3$, $\frac{3}{3} = 1$, $3^4 = 81$

$= \frac{1}{\log 3} \times \frac{\log 3^4}{1}$

$= \frac{1}{\log 3} \times \frac{4 \times \log 3}{1}$

$= 4$

Q. $\log\left(\frac{2}{3}\right)^{a} + \log\left(\frac{4}{5}\right)^{b} - \log\left(\frac{8}{15}\right)$

$\Rightarrow$

$= \log\left(\frac{2}{3} \times \frac{4}{5}\right) - \log \frac{8}{15}$ —— $\log(a \times b) = \log a + \log b$

$= \log\left(\frac{8}{15}\right) - \log\left(\frac{8}{15}\right)$

$= \log\left(\frac{8/15}{8/15}\right)$ —— $\log\left(\frac{a}{b}\right) = \log a - \log b$

$= \log 1$

$= 0$

Q. $\log_3 27 = x$

$y = a^x$

$x = \log_a y$

$a = 3$

$y = 27$

$x = x$

$27 = 3^{x}$

$27 = 3^{3}$

$x = 3$ //

Q. $\log_3 (x+6) = 2$

$y = a^x$

$x = \log_a y$

$a = 3$

$y = (x+6)$

$x = 2$

$(x+6) = 3^2$

$x + 6 = 9$

$x = 9 - 6$

$x = 3$ //

Q. find value of x

if $\log_5 (x^2 - 5x + 11) = 1$.

$\Rightarrow$

$$\log_a a = 1$$

$$\log_5 5 = 1$$

$$x^2 - 5x + 11 = 5$$

$$x^2 - 5x + 11 - 5 = 0$$

$$x^2 - 5x + 6 = 0$$

$a = 1$, $b = -5$, $c = 6$

$x_1 = 3$, $x_2 = 2$ //

Q. find value of x

if $\log_5 (x^2 - 5x + 11) = 0$.

$\Rightarrow$

$$\log_5 1 = 0.$$

$$x^2 - 5x + 11 = 1$$

$$x^2 - 5x + 11 - 1 = 0$$

$$x^2 - 5x + 10 = 0$$

$a = 1$, $b = -5$, $c = 10$

$x_1 = 2.5 + 1.936i$

$x_2 = 2.5 - 1.936i$

Partial fraction

Case I

$$\frac{ax+b}{(x-\alpha)(x-\beta)(x-\gamma)} = \frac{A}{(x-\alpha)} + \frac{B}{(x-\beta)} + \frac{C}{(x-\gamma)}$$

$$ax+b = A(x-\beta)(x-\gamma) + B(x-\alpha)(x-\gamma) + C(x-\alpha)(x-\beta) \quad \text{---} \ (1)$$

Case 2

$$\frac{ax+b}{(x-\alpha)(x-\beta)(x-\beta)} = \frac{ax+b}{(x-\alpha)(x-\beta)^2} = \frac{A}{(x-\alpha)} + \frac{B}{(x-\beta)} + \frac{C}{(x-\beta)^2}$$

$$ax+b = A(x-\beta)^2 + B(x-\alpha)(x-\beta) + C(x-\alpha) \quad \text{---} \ (1)$$

$$\frac{A\cancel{(x-\alpha)}(x-\beta)^2}{\cancel{(x-\alpha)}} + \frac{B(x-\alpha)(x-\beta)^{\cancel{2}}}{\cancel{(x-\beta)}} + \frac{C(x-\alpha)\cancel{(x-\beta)^2}}{\cancel{(x-\beta)^2}}$$

Case 3

$$\frac{ax+b}{(x-\alpha)(x^2+\beta)} = \frac{A}{(x-\alpha)} + \frac{Bx+C}{(x^2+\beta)}$$

$$ax+b = A(x^2+\beta) + (Bx+C)(x-\alpha) \quad \text{---} \ (1)$$

Q. Resolve $\frac{3x-2}{(x+2)(x^2+4)}$

$\Rightarrow \frac{3x-2}{(x+2)(x^2+4)} = \frac{A}{(x+2)} + \frac{(Bx+C)}{(x^2+4)}$

$3x-2 = A(x^2+4) + (Bx+C)(x+2)$ —— (1)

Trick $\left.\begin{matrix} x+2=0 \\ x=-2 \end{matrix}\right\}$ eq^n(1) $\Rightarrow 3(-2)-2 = A[(-2)^2+4] + [B(-2)+C][(-2)+2]$ (= 0)

$-8 = A(8) + 0$

$-8 = A(8)$

$\frac{-8}{8} = A \Rightarrow \boxed{A=-1}$

$3x-2 = A(x^2+4) + (Bx+C)(x+2)$ —— (1)

$x=0\}$ — eq^n(1) $\Rightarrow 3(0)-2 = A(0^2+4) + (B\times 0+C)(0+2)$

$-2 = (-1)(4) + C(2)$

$-2 = -4 + C\times 2$

$+4-2 = C\times 2$

$2 = C\times 2$

$\frac{2}{2} = C \Rightarrow \boxed{C=1}$

$3x-2 = A(x^2+4) + (Bx+C)(x+2)$ —— (1)

$x=1 \Rightarrow$ eq^n(1) $\Rightarrow 3\times 1-2 = A(1^2+4) + (B\times 1+C)(1+2)$

$1 = (-1)(5) + (B+1)\times 3$

$1 = -5 + (B+1)\times 3$

$+5+1 = (B+1)\times 3$

$6 = (B+1)\times 3$

$\frac{6}{3} = B+1$

$2 = B+1$

$2-1 = B$

$\boxed{1=B}$

$\frac{3x-2}{(x+2)(x^2+4)} = \frac{A}{(x+2)} + \frac{(Bx+C)}{(x^2+4)}$

$= \frac{(-1)}{x+2} + \frac{(1x+1)}{(x^2+4)}$

Q. Solve $\frac{3x-2}{(x+2)(x+4)^2}$

$\Rightarrow \frac{3x-2}{(x+2)(x+4)^2} = \frac{A}{(x+2)} + \frac{B}{(x+4)} + \frac{C}{(x+4)^2}$

$3x-2 = A(x+4)^2 + B(x+2)(x+4) + C(x+2)$ —— ①

$\left.\begin{array}{l} x+2=0 \\ x=-2 \end{array}\right\}$ eqⁿ① $\Rightarrow 3(-2)-2 = A[(-2)+4]^2 + B[(-2)+2][(-2)+4] + C[(-2)+2]$

$-8 = A[2]^2 + 0 + 0$

$-8 = A(4)$

$\frac{-8}{4} = A \Rightarrow \boxed{A=-2}$

$3x-2 = A(x+4)^2 + B(x+2)(x+4) + C(x+2)$ —— ①

$\left.\begin{array}{l} x+4=0 \\ x=-4 \end{array}\right\}$ eqⁿ① $\Rightarrow 3(-4)-2 = A[(-4)+4]^2 + B[(-4)+2][(-4)+4] + C[(-4)+2]$

$\therefore -14 = 0 + 0 + C(-2)$

$\therefore -14 = C(-2)$

$\therefore \frac{-14}{-2} = C \Rightarrow \boxed{C=7}$

$3x-2 = A(x+4)^2 + B(x+2)(x+4) + C(x+2)$ —— ①

let $x=0$, eqⁿ① $\Rightarrow 3\times 0-2 = A(0+4)^2 + B(0+2)(0+4) + C(0+2)$

$-2 = A(16) + B\times 8 + C\times 2$

$-2 = (-2)(16) + B\times 8 + 7\times 2$

$\therefore \boxed{B=2}$

$$\frac{3x-2}{(x+2)(x+4)^2} = \frac{-2}{(x+2)} + \frac{2}{(x+4)} + \frac{7}{(x+4)^2}$$

Solve $\frac{3x-2}{(x+2)(x+4)}$

$$\Rightarrow \frac{3x-2}{(x+2)(x+4)} = \frac{A}{(x+2)} + \frac{B}{(x+4)}$$

$$3x-2 = A(x+4) + B(x+2) \quad \text{—— (1)}$$

$x+4=0$, $x=-4$ } eqⁿ (1) $\Rightarrow$

$$3(-4)-2 = A[(-4)+4] + B[(-4)+2]$$

$$-14 = 0 + B(-2)$$

$$-14 = B(-2)$$

$$\frac{-14}{-2} = B \Rightarrow \boxed{B=7}$$

$$3x-2 = A(x+4) + B(x+2) \quad \text{—— (1)}$$

$x+2=0$, $x=-2$ } eqⁿ (1) $\Rightarrow$

$$3(-2)-2 = A[(-2)+4] + B[(-2)+2]$$

$$-8 = A[2] + 0$$

$$-8 = A(2)$$

$$\therefore A = \frac{-8}{2} = -4$$

$$\frac{3x-2}{(x+2)(x+4)} = \frac{A}{(x+2)} + \frac{B}{(x+4)}$$

$$= \frac{-4}{(x+2)} + \frac{7}{(x+4)}$$

Function

$$f(x) = x^2+4$$
$$= 2x^2+3x+4$$
$$= 2\cos x+\sin x+x^2$$

Function Even or Odd

$f(x) \Rightarrow$

$f(-x) = f(x)$ — even function

$f(-x) = -f(x)$ — odd function

$f(-x) \neq f(x)$, $\neq -f(x)$ — Neither even Nor odd

$\sin x \Rightarrow \sin(-x) = -\sin x$

$\cos x \Rightarrow \cos(-x) = \cos x$

$\tan x \Rightarrow \tan(-x) = -\tan x$

$\cot x \Rightarrow \cot(-x) = -\cot x$

$\sec x \Rightarrow \sec(-x) = \sec x$

$\operatorname{cosec} x \Rightarrow \operatorname{cosec}(-x) = -\operatorname{cosec} x$

state function is even or odd

$f(x) = \pm x^3 \pm 4x \pm \sin x$

$\Rightarrow$

$f(-x) = (-x)^3 + 4(-x)^1 + \sin(-x)$

$= -x^3 - 4x + (-\sin x)$

$= -x^3 - 4x - \sin x$

$f(-x) = -f(x)$

$\hookrightarrow$ $\because$ function is odd

$f(x) = +x^2 - 2x^4 + \cos x$

$\Rightarrow f(-x) = (-x)^2 - 2(-x)^4 + \cos(-x)$

$= x^2 - 2x^4 + \cos x$

$f(-x) = f(x)$

$\hookrightarrow$ function is even //

$f(x) = x^4 - 2x + 3$

find value of $f(0) + f(3) = ?$

$\Rightarrow$ $f(x) = x^4 - 2x + 3$

$f(0) = 0^4 - 2\times 0 + 3$

$= 3$

$f(3) = 3^4 - 2\times 3 + 3$

$= 78$

$f(0) + f(3) = 3 + 78$

$= 81$

$f(x) = x^4 - 2x + 3$

prove that

$f(3) = 26\, f(0)$

$\Rightarrow$ L.H.S $= f(3)$

$= 3^4 - 2\times 3 + 3$

$= 78$

RHS $= 26\, f(0)$

$= 26 \times [0^4 - 2\times 0 + 3]$

$= 26 \times 3$

$= 78$

LHS = RHS

$\therefore f(3) = 26\, f(0)$

$x^2 + 2y^2 - xy^2 = 8$ find $\frac{dy}{dx}$ at $(2,-1)$

$\Rightarrow$ differentiating w.r.t x

$$\frac{d}{dx}(x^2 + 2y^2 - xy^2) = \frac{d}{dx} 8$$

$$\frac{d}{dx} x^2 + \frac{d}{dx} 2y^2 - \frac{d}{dx} xy^2 = 0$$

$$2x + 2\frac{d}{dx} y^2 - \left[\frac{d}{dx} u \cdot v\right] = 0 \quad (u = x,\ v = y^2)$$

$$\therefore 2x + 2\left[2X \cdot \frac{d}{dx} X\right] - \left[u \cdot \frac{d}{dx} v + v \frac{d}{dx} u\right] = 0$$

$$\therefore 2x + 2\left[2y \cdot \frac{d}{dx} y\right] - \left[x \cdot \frac{d}{dx} y^2 + y^2 \cdot \frac{d}{dx} x\right] = 0$$

$$\therefore 2x + 4y \cdot \frac{dy}{dx} - \left[x \cdot 2y \cdot \frac{dy}{dx} + y^2 (1)\right] = 0$$

$$\therefore 2x + 4y \cdot \frac{dy}{dx} - 2xy \frac{dy}{dx} - y^2 = 0$$

$$4y \frac{dy}{dx} - 2xy \frac{dy}{dx} = -2x + y^2$$

$$\frac{dy}{dx}(4y - 2xy) = -2x + y^2$$

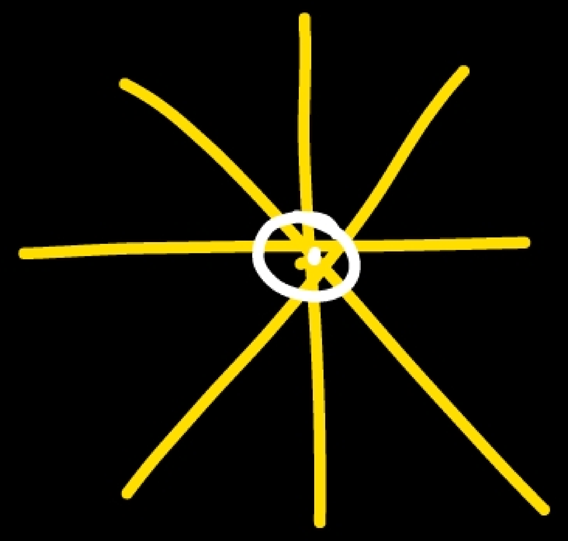
$$\therefore \frac{dy}{dx} = \frac{-2x + y^2}{4y - 2xy}$$

$$\left.\frac{dy}{dx}\right|_{at\,(2,-1)} = \frac{-2 \times 2 + (-1)^2}{4(-1) - 2 \times 2 \times (-1)}$$

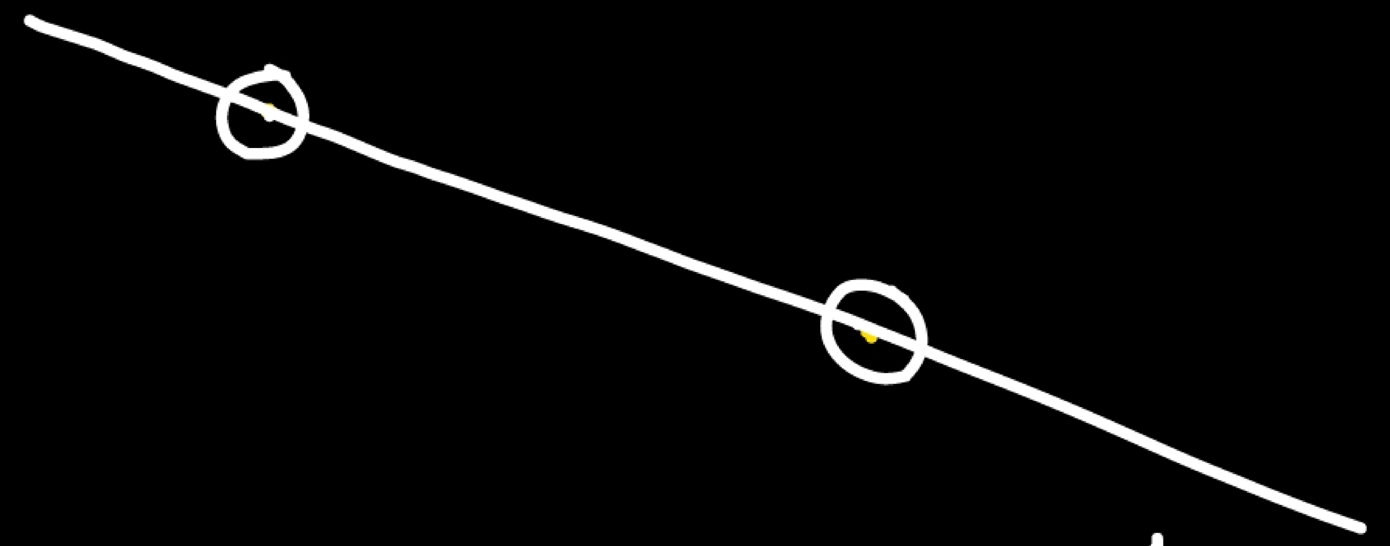
$$= \frac{-3}{0} = \infty$$

Co-ordinate geometry (12 Marks) - I-scheme

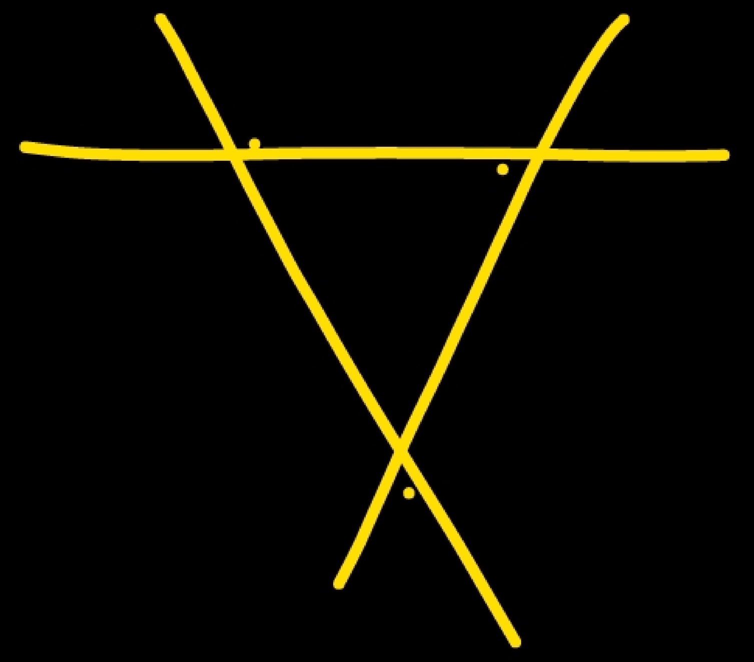
Stright line - (8 Marks) — K-scheme



∞-lines



$Ax+By+C=0$

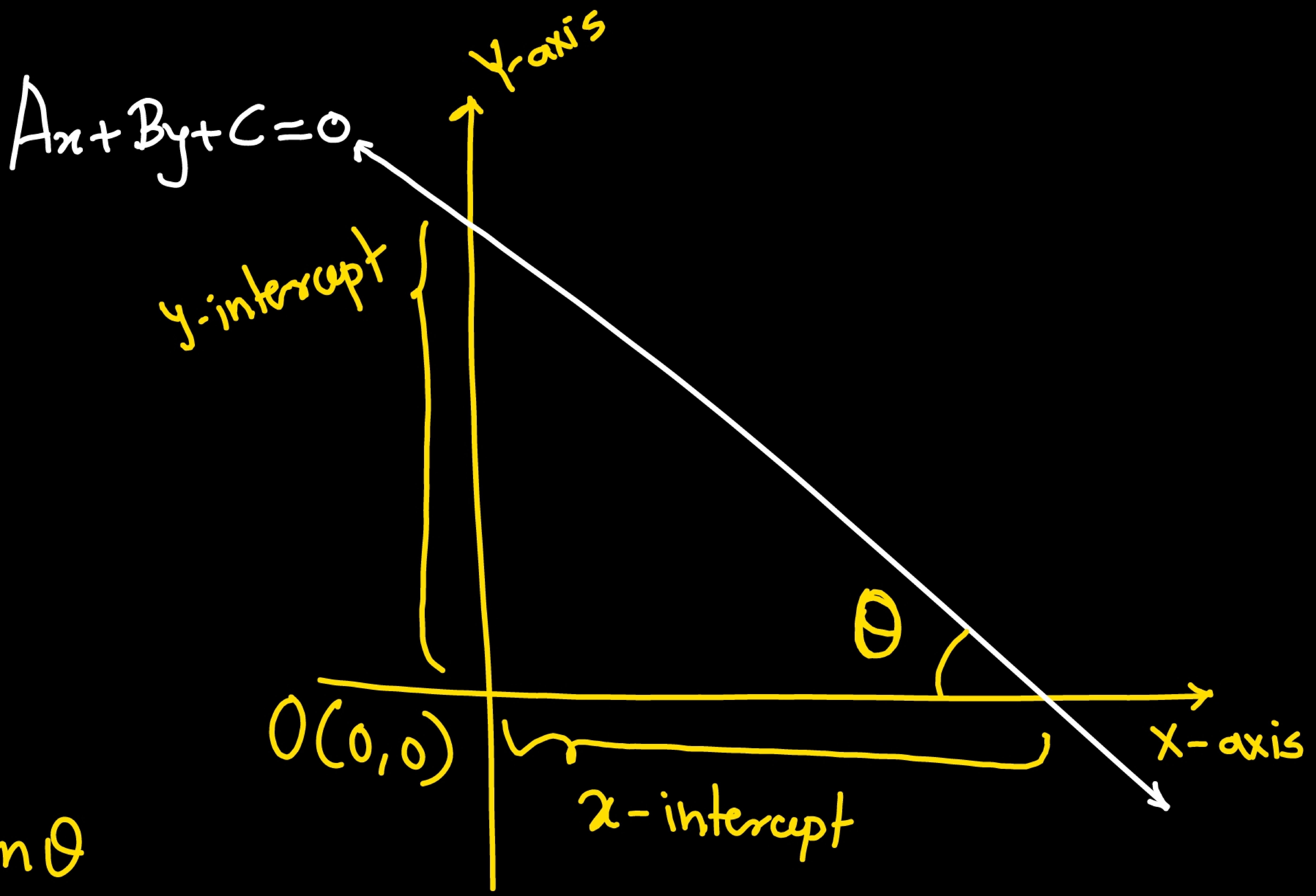


St. line eqn ⇒ $Ax + By + C = 0$

✓ $x\text{-intercept} = \dfrac{-C}{A}$

✓ $y\text{-intercept} = \dfrac{-C}{B}$

✓ $\text{slope} = \dfrac{-A}{B} = m = \tan\theta$



① One point form — a) Point (x_1, y_1)
b) Slope (m)

Eqn of straight line

$$(y - y_1) = m(x - x_1)$$

② Two point form — a) Point 1 (x_1, y_1)
b) Point 2 (x_2, y_2)

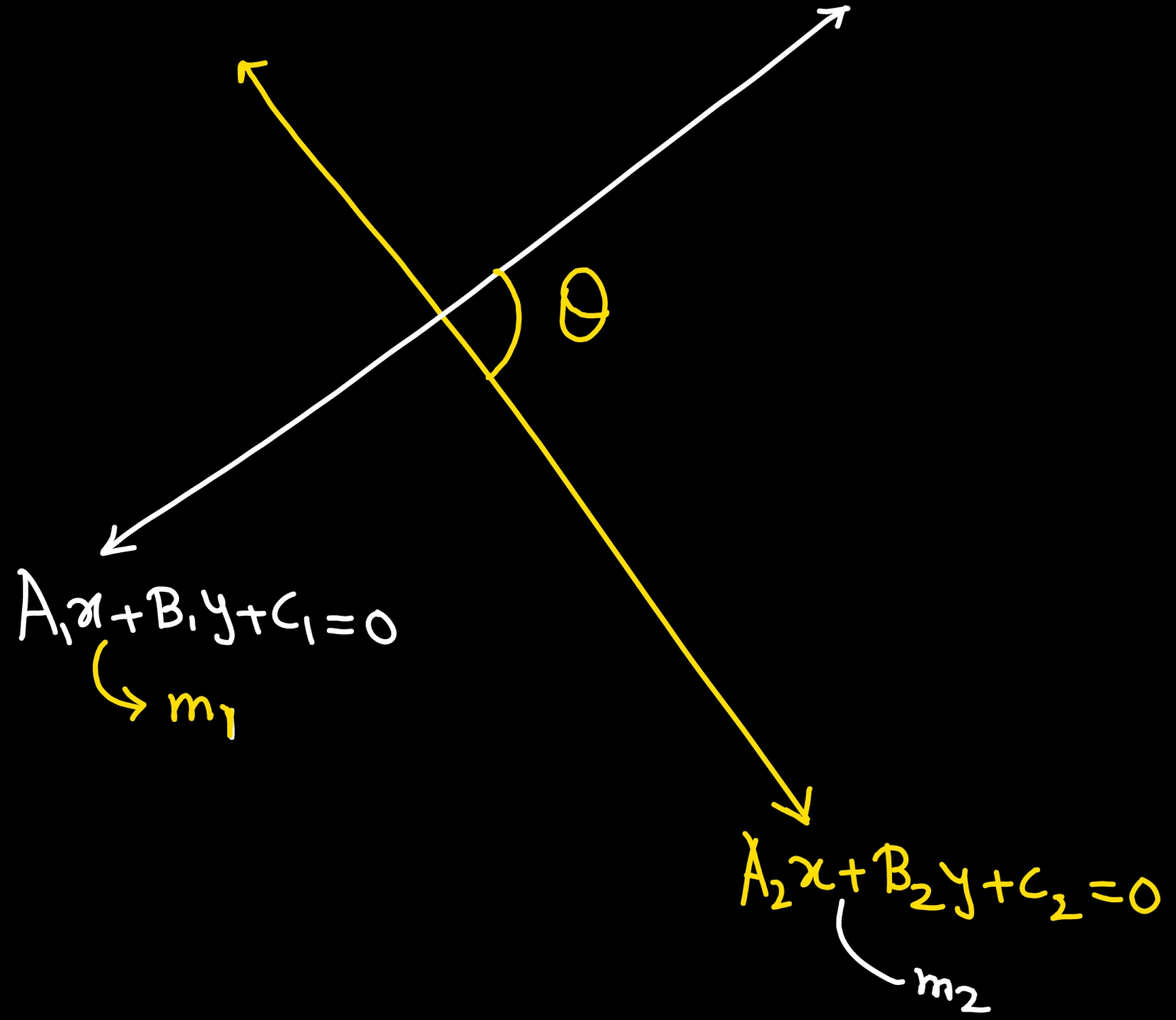
Equation of straight line

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

③ Intercept form — x-intercept $= a$
y-intercept $= b$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 \cdot m_2}\right|$$



1) If two lines are parallel

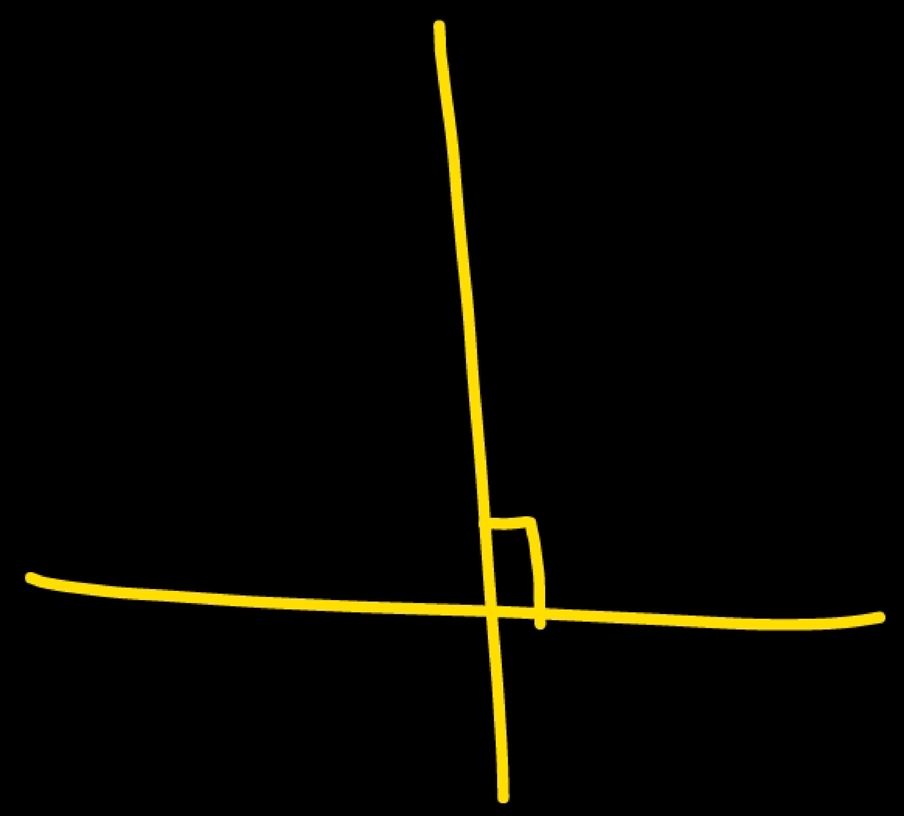
$$\theta = 0^\circ$$

$$m_1 = m_2$$

2) If two lines are perpendicular

$$\theta = 90^\circ$$

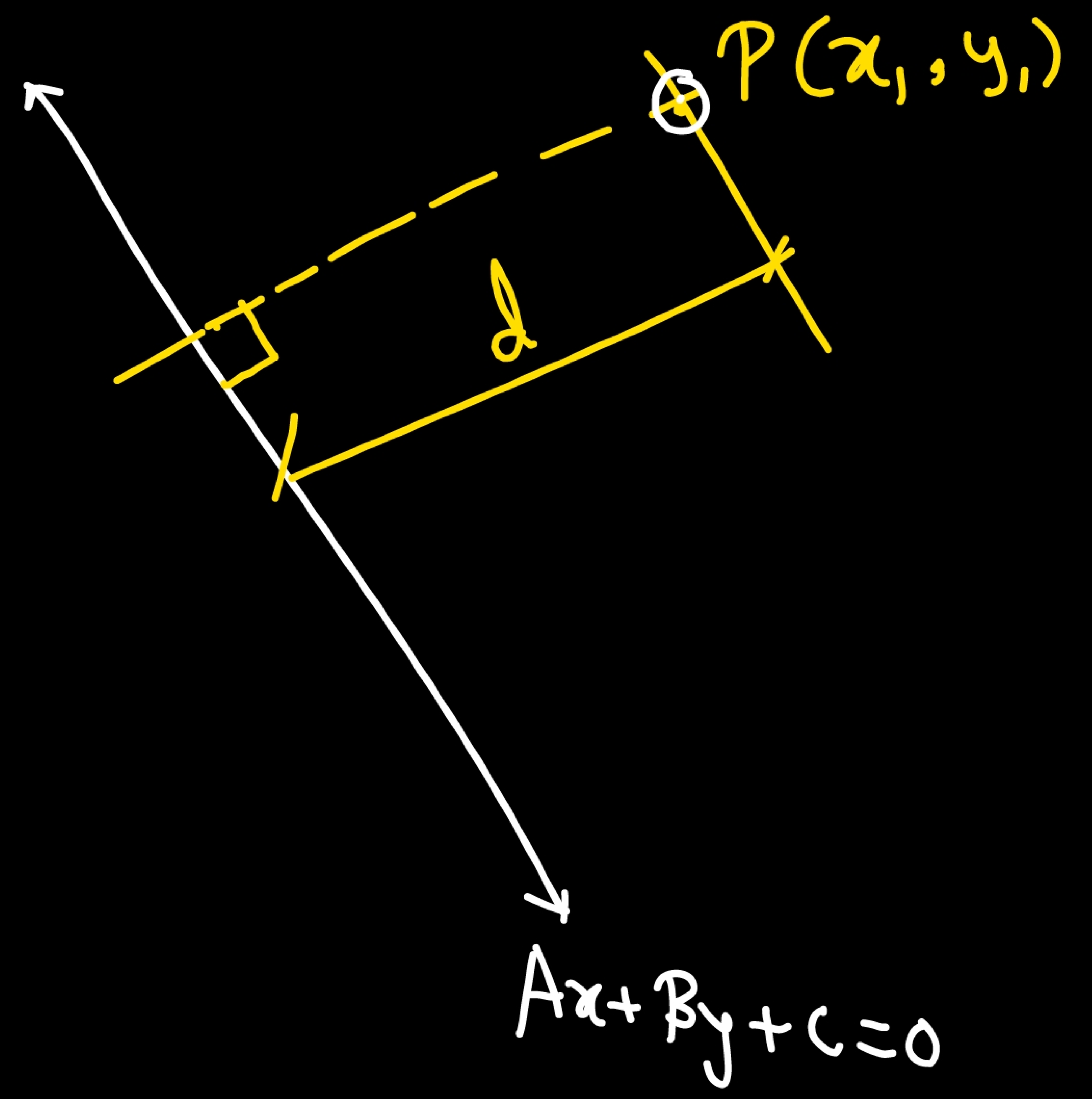
$$m_1 \times m_2 = -1$$



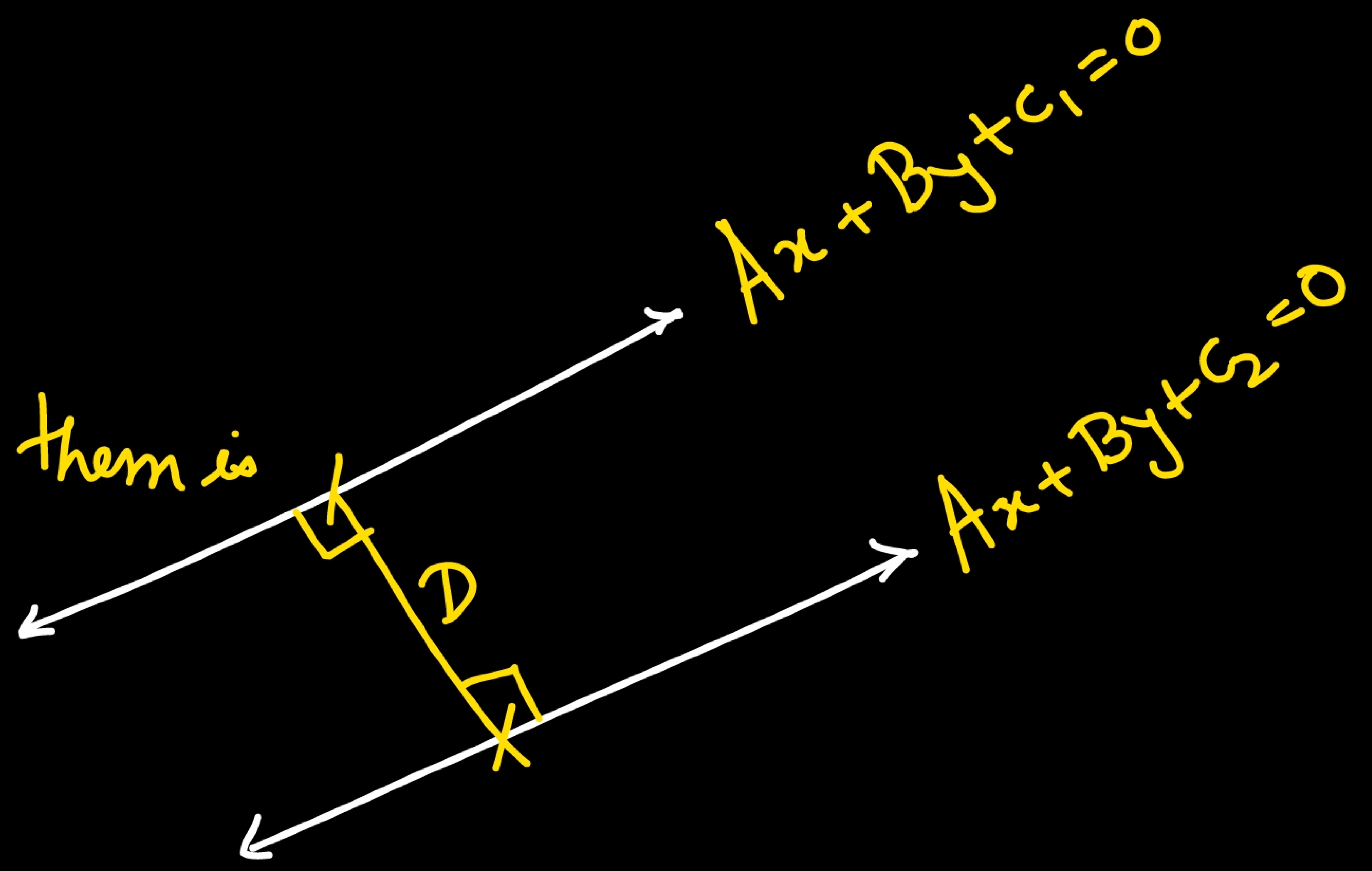
⊛ Perpendicular distance from point to the line

3 Marks

$$d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right| \text{ units}$$



⊛ If two lines are parallel then distance D between them is



$$D = \left| \frac{c_1 - c_2}{\sqrt{A^2 + B^2}} \right| \text{ units}$$

2 Marks

Q1 find intercept of line $2x + 3y = 6$ on both axes.

$\Rightarrow$

$$2x + 3y = 6$$

$$2x + 3y - 6 = 0$$

$$\ast \; Ax + By + C = 0$$

$A = 2$, $B = 3$ & $C = -6$

$$x\text{-intercept} = \frac{-C}{A} = \frac{-(-6)}{2} = \frac{6}{2} = 3 \text{ units}$$

$$y\text{-intercept} = \frac{-C}{B} = \frac{-(-6)}{3} = \frac{6}{3} = 2 \text{ units} \; //$$

4 Marks

Q. Find eq^n of straight line passing through intersection of lines $4x+3y=8$ & $x+y=1$ & parallel to $5x-7y=3$

⇒

$4x+3y=8$ —— ①

$1x+y=1$ —— ②

Step ① Find point of intersection

coeff of x in eq^n ②

eq^n ① × [1] ⇒ $4x+3y=8$

eq^n ② × [4] ⇒ $4x+4y=4$

coeff of x in eq^n ①

$0-y=4$

$-y=4$

$y=\frac{4}{-1}=-4$

∴ $y=-4$

eq^n ② ⇒ $x+y=1$

$x+(-4)=1$

$x-4=1$

$x=1+4$

$x=5$

∴ Point of intersection $=(x,y)=(5,-4)$

Step ② Find slope of line parallel to $5x-7y=3$

- ① $\theta=0°$
- ② $m_1=m_2$

$5x-7y=3$

$5x-7y-3=0$

$Ax+By+C=0$

$A=5$, $B=-7$ & $C=-3$

$\text{Slope}=m_2=\frac{-A}{B}$

$=\frac{-(5)}{-7}$

$m_2=0.7143$

Step ③ Eq^n of st. line
- a) Point $(x_1,y_1)=(5,-4)$
- b) Slope $m_2=0.7143$

eq^n of st. line

$(y-y_1)=m_2(x-x_1)$

$[y-(-4)]=0.7143(x-5)$

$y+4=0.7143x-5\times0.7143$

$y+4=0.7143x-3.5715$

$-0.7143x+3.5715+y+4=0$

$-0.7143x+y+7.5715=0$

Q.5 c (i) find distance between two parallel lines

$$3x + 2y = 5 \quad \& \quad 3x + 2y = 6$$

⇒

$$Ax + By + C = 0$$

$3x + 2y = 5$ $3x + 2y = 6$

$3x + 2y - 5 = 0$ $3x + 2y - 6 = 0$

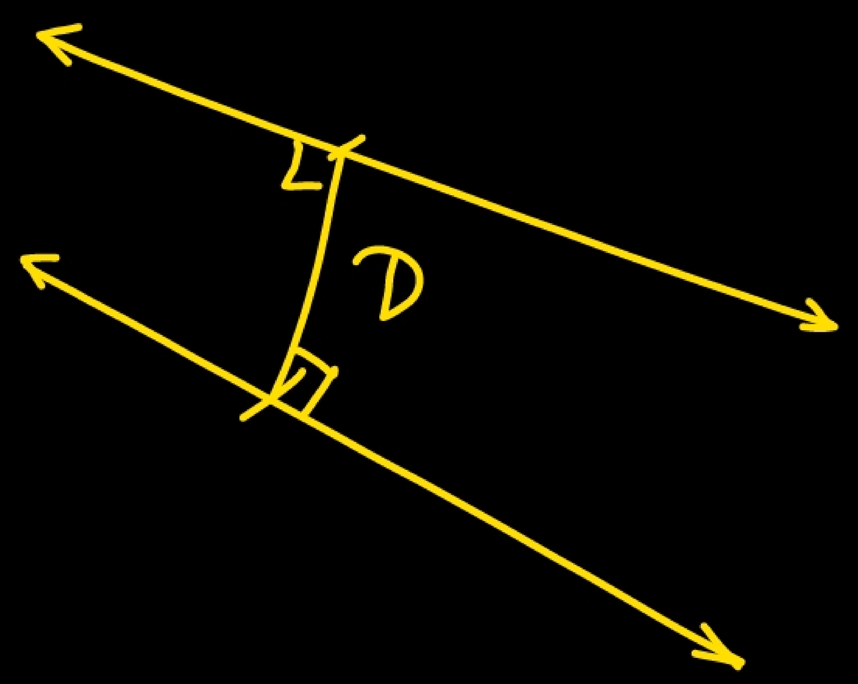
$Ax + By + C_1 = 0$ ① $Ax + By + C_2 = 0$ ②

⇒ $C_2 = -6$

$C_1 = -5$

$A = 3$

$B = 2$



$$D = \left| \frac{C_1 - C_2}{\sqrt{A^2 + B^2}} \right| \text{ units}$$

$$= \left| \frac{(-5) - (-6)}{\sqrt{3^2 + 2^2}} \right|$$

$$= \left| \frac{1}{\sqrt{9+4}} \right| = 0.2774 \text{ units}$$

Q. Find angle between line

$3x = y - 4$ and

$2x + y + 3 = 0$

$\Rightarrow$

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 \cdot m_2}\right|$$

$3x = y - 4$

$3x - y + 4 = 0$

$Ax + By + C = 0$

$A = 3,\ B = -1,\ C = 4$

$$\text{slope} = m_1 = \frac{-A}{B}$$

$$m_1 = \frac{-3}{-1} = 3$$

$\boxed{m_1 = 3}$

$2x + y + 3 = 0$

$Ax + By + C = 0$

$A = 2,\ B = 1\ \&\ C = 3$

$$m_2 = \frac{-A}{B} = \frac{-2}{1}$$

$\therefore \boxed{m_2 = -2}$

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 \times m_2}\right|$$

$$= \tan^{-1}\left|\frac{3 - (-2)}{1 + 3 \times (-2)}\right|$$

$$= \tan^{-1}\left|\frac{5}{-5}\right|$$

$$= \tan^{-1}|-1|$$

$$= \tan^{-1}(1)$$

$$\theta = 45^\circ$$

//

Q. Find eq^n of st. line passing through $(-3, 10)$ & addition of intercept is 8

⇒ step ① Eq^n of st. line

$$y = mx + c$$

x-intercept ⇒ $y = 0$ ⇒ $0 = mx + c$

$\therefore -c = mx$

$\therefore \frac{-c}{m} = x\text{-intercept}$

y-intercept ⇒ $x = 0$ ⇒ $y = m \times 0 + c$

$y = c = y\text{-intercept}$

Addition of intercept is 8

$\therefore x\text{-int} + y\text{ int} = 8$

$$\frac{-c}{m} + c = 8 \quad \text{——} ①$$

Line is passing through $(-3, 10)$

$y = mx + c$

$10 = m(-3) + c$

$10 = -3m + c \quad \text{——} ②$

$\therefore c = 10 + 3m$

eq^n ① ⇒ $\frac{-c}{m} + c = 8$

$$-\frac{(10+3m)}{m} + \frac{(10+3m)}{1} = 8$$

$$\frac{-(10+3m) + m(10+3m)}{m} = 8$$

$$-10 - 3m + m(10 + 3m) = 8m$$

$$-10 - 3m + 10m + 3m^2 - 8m = 0$$

EQN Degree ② ⟵ $3m^2 - m - 10 = 0$

$a = 3$
$b = -1$
$c = -10$

$m_1 = 2$ OR $m_2 = -1.667$

$c_1 = 10 + 3m$
$= 10 + 3 \times 2$
$c_1 = 16$

$y = mx + c$

$y = 2x + 16$

$-2x + y - 16 = 0$ —— ①

$c_2 = 10 + 3m$
$= 10 + 3(-1.667)$
$= 4.999$
$c_2 \simeq 5$

$y = mx + c$

$y = -1.667x + 5$

$1.667x + y - 5 = 0$ —— ②

$$\frac{a}{0} = \infty$$

$$\frac{0}{a} = 0$$

$$a^0 = 1$$

$$\tan^{-1}\left(\frac{x}{0}\right)$$

$$\tan^{-1}(\infty)$$

$$\theta = \tan^{-1}(\infty)$$

$$\tan\theta = \infty$$

$$\tan 90^\circ = \text{Math. error} \quad \infty$$

$$\tan 90^\circ = \infty$$

$$\therefore \theta = 90^\circ$$ //

Matrix

4 Marks

Q. If $A=\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$ & $B=\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$

then find X such that

$$2X+3A-4B=I$$

Where I is identity matrix of order 2

⇒

$$2X+3A-4B=I$$

$$\therefore 2X+3\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}-4\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore 2X+\begin{bmatrix} 9 & -3 \\ 6 & 12 \end{bmatrix}-\begin{bmatrix} 4 & 8 \\ -12 & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore 2X+\begin{bmatrix} 9-4 & -3-8 \\ 6-(-12) & 12-0 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore 2X+\begin{bmatrix} 5 & -11 \\ 18 & 12 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore 2X=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}-\begin{bmatrix} 5 & -11 \\ 18 & 12 \end{bmatrix}$$

$$2X=\begin{bmatrix} 1-5 & 0-(-11) \\ 0-18 & 1-12 \end{bmatrix}$$

$$2X=\begin{bmatrix} -4 & 11 \\ -18 & -11 \end{bmatrix}$$

$$X=\frac{1}{2}\begin{bmatrix} -4 & 11 \\ -18 & -11 \end{bmatrix}$$

$$=\begin{bmatrix} -4/2 & 11/2 \\ -18/2 & -11/2 \end{bmatrix}$$

$$X=\begin{bmatrix} -2 & 5.5 \\ -9 & -5.5 \end{bmatrix}$$

//

Q. State whether AB is singular Matrix or Not?

if $A = \begin{bmatrix} -2 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 1 \\ 3 & 5 \\ 0 & 2 \end{bmatrix}$

⇒

$AB = \begin{bmatrix} -2 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}_{2\times3} \times \begin{bmatrix} 2 & 1 \\ 3 & 5 \\ 0 & 2 \end{bmatrix}_{3\times2}$

$\therefore AB = \begin{bmatrix} -2\times2+0\times3+2\times0 & -2\times1+0\times5+2\times2 \\ 3\times2+4\times3+5\times0 & 3\times1+4\times5+5\times2 \end{bmatrix}$

$AB = \begin{bmatrix} -4 & 2 \\ 18 & 33 \end{bmatrix}$

$\|AB\| = 0$	— singular Matrix
$\|AB\| \neq 0$	— Non-singular Matrix

$|AB| = \begin{vmatrix} -4 & 2 \\ 18 & 33 \end{vmatrix}$

$= (-4)\times33 - 18\times2$

$= -168 \neq 0$ — hence AB is Non-singular Matrix

6 Marks

Q. Solve by Matrix Inversion method

$x+y+z=3$; $3x-2y+3z=4$ & $5x+5y+z=11$

⇒

$$1x + 1y + 1z = 3$$
$$3x - 2y + 3z = 4$$
$$5x + 5y + 1z = 11$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -2 & 3 \\ 5 & 5 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 11 \end{bmatrix}$$

$$[A].[X] = [B]$$
$$[X] = \frac{[B]}{[A]} = \frac{1}{[A]} \cdot [B]$$
$$[X] = [A]^{-1} \cdot [B]$$
$$[A]^{-1} = \frac{1}{|A|} \cdot \text{Adj } A$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 3 & -2 & 3 \\ 5 & 5 & 1 \end{vmatrix} = 1 \begin{vmatrix} -2 & 3 \\ 5 & 1 \end{vmatrix} - 1 \begin{vmatrix} 3 & 3 \\ 5 & 1 \end{vmatrix} + 1 \begin{vmatrix} 3 & -2 \\ 5 & 5 \end{vmatrix}$$

$$= 1[(-2)\times1 - 5\times3] - 1[3\times1 - 5\times3] + 1[3\times5 - 5\times(-2)]$$
$$|A| = 20 \neq \quad [A]^{-1} \text{ exists}$$

$$C_{11} = (-1)^{1+1}\begin{vmatrix} -2 & 3 \\ 5 & 1 \end{vmatrix} = (-1)^2[(-2)\times1 - 5\times3] = -17$$

$$C_{12} = (-1)^{1+2}\begin{vmatrix} 3 & 3 \\ 5 & 1 \end{vmatrix} = (-1)^3[3\times1 - 5\times3] = 12$$

$$C_{13} = (-1)^{1+3}\begin{vmatrix} 3 & -2 \\ 5 & 5 \end{vmatrix} = (-1)^4[3\times5 - 5\times(-2)] = 25$$

$$C_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} = (-1)^3[1\times1 - 5\times1] = 4$$

$$C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} = (-1)^4[1\times1 - 5\times1] = -4$$

$$C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 5 & 5 \end{vmatrix} = (-1)^5[1\times5 - 5\times1] = 0$$

$$C_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix} = (-1)^4[1\times3 - (-2)\times1] = 5$$

$$C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} = (-1)^5[1\times3 - 1\times3] = 0$$

$$C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} = (-1)^6[1\times(-2) - 3\times1] = -5$$

$$\therefore \text{Co-factor Matrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} -17 & 12 & 25 \\ 4 & -4 & 0 \\ 5 & 0 & -5 \end{bmatrix}$$

$$\text{Adj } A = [C]^T = \begin{bmatrix} -17 & 4 & 5 \\ 12 & -4 & 0 \\ 25 & 0 & -5 \end{bmatrix}$$

$$[A]^{-1} = \frac{1}{|A|} \cdot \text{Adj } A$$
$$[A]^{-1} = \frac{1}{20}\begin{bmatrix} -17 & 4 & 5 \\ 12 & -4 & 0 \\ 25 & 0 & -5 \end{bmatrix}$$

$$[X] = [A]^{-1}[B]$$
$$= \frac{1}{20}\begin{bmatrix} -17 & 4 & 5 \\ 12 & -4 & 0 \\ 25 & 0 & -5 \end{bmatrix}_{3\times3} \begin{bmatrix} 3 \\ 4 \\ 11 \end{bmatrix}_{3\times1}$$

$$= \frac{1}{20}\begin{bmatrix} -17\times3 + 4\times4 + 5\times11 \\ 12\times3 + (-4)\times4 + 0\times11 \\ 25\times3 + 0\times4 + (-5)\times11 \end{bmatrix}$$

$$= \frac{1}{20}\begin{bmatrix} 20 \\ 20 \\ 20 \end{bmatrix}$$

$$[X] = \begin{bmatrix} 20/20 \\ 20/20 \\ 20/20 \end{bmatrix}$$

$$[X] = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$x = 1, \quad y = 1, \quad z = 1$$

I-scheme 4 Marks

Q. Solve using Cramer's Rule

$x + y + z = 3$

$3x - 2y + 3z = 4$

$5x + 5y + z = 11$

⇒

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 3 & -2 & 3 \\ 5 & 5 & 1 \end{vmatrix} = 1\begin{vmatrix} -2 & 3 \\ 5 & 1 \end{vmatrix} - 1\begin{vmatrix} 3 & 3 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & -2 \\ 5 & 5 \end{vmatrix}$$

$$= 1[(-2)\times1 - 5\times3] - 1[3\times1 - 5\times3] + 1[3\times5 - 5\times(-2)] = -17 - 1(-12) + 25 = 20$$

$D = 20$

$$D_x = \begin{vmatrix} 3 & 1 & 1 \\ 4 & -2 & 3 \\ 11 & 5 & 1 \end{vmatrix} = 3\begin{vmatrix} -2 & 3 \\ 5 & 1 \end{vmatrix} - 1\begin{vmatrix} 4 & 3 \\ 11 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & -2 \\ 11 & 5 \end{vmatrix}$$

$$= 3[(-2)\times1 - 5\times3] - 1[4\times1 - 11\times3] + 1[4\times5 - 11\times(-2)] = -51 + 29 + 42$$

$\therefore D_x = 20$

$$D_y = \begin{vmatrix} 1 & 3 & 1 \\ 3 & 4 & 3 \\ 5 & 11 & 1 \end{vmatrix} = 1\begin{vmatrix} 4 & 3 \\ 11 & 1 \end{vmatrix} - 3\begin{vmatrix} 3 & 3 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 4 \\ 5 & 11 \end{vmatrix}$$

$$= 1[4\times1 - 11\times3] - 3[3\times1 - 5\times3] + 1[3\times11 - 5\times4] = -29 + 36 + 13$$

$D_y = 20$

$$D_z = \begin{vmatrix} 1 & 1 & 3 \\ 3 & -2 & 4 \\ 5 & 5 & 11 \end{vmatrix} = 1\begin{vmatrix} -2 & 4 \\ 5 & 11 \end{vmatrix} - 1\begin{vmatrix} 3 & 4 \\ 5 & 11 \end{vmatrix} + 3\begin{vmatrix} 3 & -2 \\ 5 & 5 \end{vmatrix}$$

$$= 1[(-2)\times11 - 5\times4] - 1[3\times11 - 5\times4] + 3[3\times5 - 5\times(-2)] = -42 - 13 + 75$$

$D_z = 20$

$D = 20$, $D_x = 20$, $D_y = 20$ & $D_z = 20$

$$x = \frac{D_x}{D} = \frac{20}{20} = 1$$

$$y = \frac{D_y}{D} = \frac{20}{20} = 1$$

$$z = \frac{D_z}{D} = \frac{20}{20} = 1$$

$x = 1$, $y = 1$ & $z = 1$

Q. Find area of triangle whose vertices are $(-8,-2)$, $(-4,-6)$ & $(-1,5)$

$\Rightarrow$ Area of triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} -8 & -2 & 1 \\ -4 & -6 & 1 \\ -1 & 5 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left\{ (-8)\begin{vmatrix} -6 & 1 \\ 5 & 1 \end{vmatrix} - (-2)\begin{vmatrix} -4 & 1 \\ -1 & 1 \end{vmatrix} + 1\begin{vmatrix} -4 & -6 \\ -1 & 5 \end{vmatrix} \right\}$$

$$= \frac{1}{2}\left\{ (-8)[(-6)\times 1 - 5\times 1] - (-2)[(-4)\times 1 - (-1)\times 1] + 1[(-4)(5) - (-1)(-6)] \right\}$$

$$= \frac{1}{2}\left\{ 88 - 6 - 26 \right\}$$

$$= \frac{1}{2}\times 56$$

Area of triangle $= 28$ sq. units //